

(Following Paper ID and Roll No. to be filled in your Answer Book)

**PAPER ID : 9929**

Roll No.

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**B.Tech.**(SEM II) EVEN SEMESTER THEORY EXAMINATION,  
2009-2010**MATHEMATICS - II**

Time : 3 Hours

Total Marks : 100

Note : (i) Attempt ALL questions.

(ii) All questions carry equal marks.

1. Attempt any four parts of the following : (4x5=20)

(a) Solve  $(2x - 4y + 5) dy + (x - 2y + 3) dx = 0$

(b) Solve the initial value problem

$$\frac{dy}{dx} + y \tan x = \sin 2x, y(0) = 1.0015$$

(c) Solve  $(D^2 - 4D + 4)y = e^x \cos x$

(d) Solve by using variation of parameters

$$x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} - 9y = 48 x^5$$

(e) Solve :

$$\frac{dx}{dt} + \frac{dy}{dt} + 2x + y = 0$$

$$\frac{dy}{dt} + 5x + 3y = 0$$

- (f) An inductance  $L$  of  $5.0$  H and a resistance  $R$  of  $25 \Omega$  are connected in series with an emf  $E$  volt. If the current  $I$  is zero when  $t=0$ , find the current  $I$  at the end of  $1$  second if  $E = 100$  V, using the differential equation

$$L \frac{dI}{dt} + IR = E$$

2. Solve any four parts of the following : (4x5=20)

- (a) Evaluate the integral using Laplace transform

$$\int_0^{\infty} e^{-t} \frac{\cos^2 t}{t} dt$$

- (b) Find Laplace transform of the function

$$f(t) = \begin{cases} \frac{t}{\omega} & , 0 < t < \omega \\ 1 - \frac{t}{\omega} & , \omega < t < 2\omega \\ 1 & 2\omega < t < \infty \end{cases}$$

- (c) State first shifting theorem for Laplace transform. Using this theorem or otherwise, find the Laplace transform of the function  $f(t) = e^{-\alpha t} \cos \beta t$

- (d) State convolution theorem for the Laplace transform. Hence or otherwise find the function whose Laplace transform is

$$\frac{s^2}{(s^2 + w^2)^2}$$

- (e) Solve :

$$\frac{d^2 x}{dt^2} + 3 \frac{dx}{dt} + 2x = r(t)$$

$$\text{Where } r(t) = \begin{cases} e^t, & 0 < t < 2 \\ 0 & t > 2 \end{cases}$$

$$\text{and } x(0) = 1, x'(0) = -2.$$

- (f) Find the Laplace transform of a periodic function  $f(t)$  with period  $\tau$

3. Attempt any two parts of the following : (2x10=20)

- (a) Solve the differential equation  $(1-x^2)y'' - 2xy' + n(n+1)y = 0$  by power series method,  $n$  being an integer.
- (b) Show that

$$P_n(x) = \frac{1}{2^n n!} \frac{d^n}{dx^n} (x^2-1)^n$$

Where  $P_n(x)$  is Legendre polynomial of degree  $n$ .

- (c) Prove the recurrence relations

(i)  $J_{n+3}(x) + J_{n+5}(x) = \frac{2}{x} (n+4) J_{n+4}(x)$

(ii)  $x J_n'(x) = n J_n(x) - x J_{n+1}(x)$

Where  $J_n(x)$  is Bessel function.

4. Attempt any two parts of the following : (2x10=20)

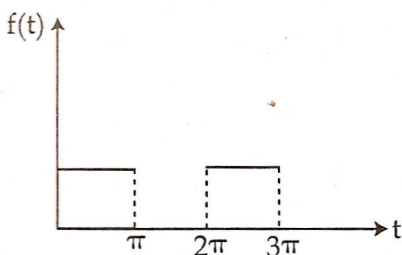
- (a) Obtain the Fourier series for function

$$f(x) = \begin{cases} x, & -\pi < x < 0 \\ -x & 0 < x < \pi, \end{cases}$$

and hence deduce that

$$\frac{\pi^2}{12} = 1 - \frac{1}{4} + \frac{1}{9} - \frac{1}{16} + \dots$$

- (b) Obtain the Fourier series for the square wave form up to 4 terms as shown in the following figure :



- (c) Give a concise description of the classification of linear partial differential equations of second order with constant coefficients. Give examples of each type of partial differential equations.

5. Attempt any two of the following :

(2x10=20)

- (a) Solve :

$$(D^2 - 4DD' + 4D'^2)z = e^{2x+y}$$

$$D \equiv \frac{\partial}{\partial x}, D' = \frac{\partial}{\partial y}$$

- (b) Describe stepwise separable of variable method to solve Laplace equation in two dim. with usual boundary / and/or initial condition.
- (c) Find the temperature  $u(x,t)$  in a slab whose ends  $x=0$  and  $x=l$  are kept at temperature zero and whose initial temperature  $f(x)$  is given by

$$f(x) = \begin{cases} A & \text{When } 0 < x < \frac{l}{2} \\ 0 & \frac{l}{2} < x < l \end{cases}$$

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