M.F

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Printed Pages – 5 TAS – 104/MA – 101/MA – 101 (O)

(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 9906
9916

Roll No.

B.Tech.

FIRST SEMESTER EXAMINATION, 2005-2006

MATHEMATICS - I

Time : 3 Hours *Total Marks : 100*

Note : *(i)* *Attempt ALL questions.*

(ii) *All questions carry equal marks.*

(iii) *Question no. 1 - 4 are common to all candidates.*

(iv) *Be precise in your answer.*

1. Attempt *any four* parts of the following : **(5x4=20)**

(a) Use elementary transformation to reduce the following matrix A to triangular form and hence find the rank of A.

$$A=\begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$$

(b) Define Unitary Matrix. Show that the matrix $\begin{bmatrix} \alpha+i\gamma & -\beta+i\delta \\ \beta+i\delta & \alpha-i\gamma \end{bmatrix}$ is a unitary matrix if $\alpha^2+\beta^2+\gamma^2+\delta^2=1$.

(c) Reduce the matrix A to diagonal form

$$A=\begin{bmatrix}-1 & 2 & -2\\ 1 & 2 & 1\\ -1 & -1 & 0\end{bmatrix}.$$

(d) Find the eigen values and eigen vectors of matrix A

$$A=\begin{bmatrix}1 & 7 & 13\\ 2 & 5 & 7\\ 3 & 11 & 5\end{bmatrix}.$$

(e) Test the consistency of following system of linear equations and hence find the solution

$$\begin{aligned}4x_1-x_2 &= 12\\ -x_1+5x_2-2x_3 &= 0\\ -2x_2+4x_3 &= -8\end{aligned}$$

(f) State Cayley-Hamilton theorem. Using this theorem find the inverse of the matrix.

$$A=\begin{bmatrix}2 & -1 & 1\\ -1 & 2 & -1\\ 1 & -1 & 2\end{bmatrix}$$

2. Attempt *any four* parts of the following : **(5x4=20)**

(a) Find the directional derivative of $\frac{1}{r^2}$ in the direction of $\vec{r}$, where $\vec{r}=\hat{i}\,x+\hat{j}\,y+\hat{k}\,z$.

(b) Find $\iint \vec{F}.\hat{n}\,ds$, where

$\vec{F}=(2x+3z)\,\hat{i}-(xz+y)\,\hat{j}+(y^2+2z)\,\hat{k}$ and s is the surface of sphere having centre (3, −1, 2) and radius 3.

(c) Show that the vector field $\vec{F} = \frac{\vec{r}}{r^3}$ is irrotational as well as solenoidal. Find the scalar potential.

(d) If $\vec{A}$ is a vector function and ϕ is a scalar function, then show that $\nabla.(\phi\overrightarrow{A}) = \phi\nabla\cdot\overrightarrow{A} + \overrightarrow{A}.\nabla\phi$.

(e) Apply Green's theorem to evaluate $\oint_C 2y^2\, dx + 3x\, dy$ where c is the boundary of closed region bounded between $y = x$ and $y = x^2$.

(f) Suppose $\vec{F}(x, y, z) = x^3\,\hat{i} + y\,\hat{j} + z\,\hat{k}$ is the force field. Find the work done by $\vec{F}$ along the line from the (1, 2, 3) to (3, 5, 7).

3. Attempt *any four* parts of the following : **(5x4=20)**

(a) If $y = (\sin^{-1}x)^2$, prove that
$(1-x^2)\, y_{n+2} - (2n+1)\, x\, y_{n+1} - n^2 y_n = 0$.
Hence find the value of y_n at $x = 0$.

(b) If $u = f(r)$ and $x = r\cos\theta$, $y = r\sin\theta$, prove that

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f''(r) + \frac{1}{r} f'(r).$$

(c) Trace the cure $x^{2/3} + y^{2/3} = a^{2/3}$.

(d) Expand $\tan^{-1}\left(\frac{y}{x}\right)$ in the neighbourhood of (1, 1).

(e) If $u = x\log(xy)$, where $x^3 + y^3 + 3xy = 1$. Find $\frac{du}{dx}$.

(f) State Euler's theorem of differential calculus. Hence verify the theorem for the function $u = \log \frac{x^2 + y^2}{xy}$.

4. Attempt *any two* parts of the following : **(10x2=20)**

(a) If J be the Jacobian of the system u, v with regard to x, y and J' the Jacobian of the system x, y with regard to u, v, then prove that $JJ' = 1$.

(b) A rectangular box open at top is to have a given capacity. Find the dimensions of the box requiring least material.

(c) A balloon is in the form of right circular cylinder of radius 1.5 m and length 4.0 m and is surmounted by hemispherical ends. If the radius is increased by 0.01 m and length by 0.05 m, find the percentage change in the volume of balloon.

FOR NEW SYLLABUS ONLY (TAS-104/MA-101)

5. Attempt *any two* parts of the following : **(10x2=20)**

(a) Evaluate the integral $\int_0^\infty \int_0^x x \exp\left(-\frac{x^2}{y}\right) dy\, dx$ by changing the order of integration.

(b) Find by triple integration, the volume of the paraboloid of revolution $x^2 + y^2 = 4z$ cut off by the plane $z = 4$.

(c) State the Dirichlet's theorem for three variables. Hence evaluate the integral

$$\iiint x^{l-1} y^{m-1} z^{n-1}\, dx\, dy\, dz.$$

where x,y,z are all positive but limited by the condition $\left(\frac{x}{a}\right)^p + \left(\frac{y}{b}\right)^q + \left(\frac{z}{c}\right)^r \le 1$.

FOR OLD SYLLABUS ONLY MA-101 (OLD)

5. Attempt *any two* parts of the following : **(10x2=20)**

(a) The following data regarding the heights (y) and weights (x) of 100 college students are given $\Sigma x = 15000$, $\Sigma x^2 = 2272500$, $\Sigma y = 6800$, $\Sigma y = 463025$ and $\Sigma xy = 1022250$.

Find the correlation coefficient between height and weight and equation of regression line of height on weight.

(b) Fit a Poisson distribution to the following data and calculate the theoretical frequencies.

x	0	1	2	3	4
f	192	100	24	3	1

(c) Assume the mean height of soldiers to be 68.22 inches with a variance of 10.8 inches square. How many soldiers in a regiment of 10,000 would you expect to be over 6 feet tall, given that the area under the standard normal curve between $x = 0$ and $x = 0.31$ is 0.1368 and between $x = 0$ and $x = 1.15$ is 0.3746.

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