(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 9916
9906
9956

Roll No. | | | | | | | | | | |

B. Tech.

FIRST SEMESTER EXAMINATION, 2006-07

MATHEMATICS - I

Time : 3 Hours *Total Marks : 100*

Note : *(i) Attempt* ***ALL*** *questions.*

(ii) All questions carry equal marks.

(iii) In case of numerical problems assume data wherever not provided.

(iv) Be precise in your answer.

1. Attempt *any four* parts of the following : **(5x4=20)**

(a) If $y = x \log(1+x)$, prove that

$$y_n = [(-1)^{n-2} \underline{|n-2} \,(x+n)]/(1+x)^n$$

(b) If x = tany, prove that

$$(1+x^2)y_{n+1} + (2nx-1)y_n + n(n-1)\,y_{n-1} = 0$$

(c) If $u(x, y, z) = \log(\tan x + \tan y + \tan z)$

prove that $\sin 2x \dfrac{\partial u}{\partial x} + \sin 2y \dfrac{\partial u}{\partial y} + \sin 2z \dfrac{\partial u}{\partial z} = 2$

(d) State and prove Euler's theorem for partial differentiation of a homogeneous function f(x, y).

(e) If $u(x, y) = \sin^{-1}\frac{x}{y} + \tan^{-1}\frac{y}{x}$

prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 0$

(f) Trace the curve $y^2(a-x) = x^3 \quad a > 0$

2. Attempt *any two* parts of the following : (10x2=20)

(a) If $u^3 + v^3 = x + y,\ u^2 + v^2 = x^3 + y^3$

show that $\frac{\partial(u,v)}{\partial(x,y)} = \frac{y^2 - x^2}{2uv(u-v)}$

(b) Find Taylor series expansion of function on $f(x,\ y) = e^{-x^2-y^2}\cos xy$ about the point $x_0 = 0,\ y_0 = 0$ up to three terms.

(c) Find the minimum distance from the point (1, 2, 0) to the cone $z^2 = x^2 + y^2$.

3. Attempt *any two* parts of the following : (10x2=20)

(a) Define the gradient, divergence and curl.

(i) If $f(x, y, z) = 3x^2y - y^3z^2$, find grad f at the point (1, −2, −1).

(ii) If $\bar{F}(x,y,z) = xz^3\hat{i} - 2x^2yz\hat{j} + 2yz^4\hat{k}$ find divergence and curl of $\vec{F}(x, y, z)$.

(c) Find the value of λ for which the vectors $(1, -2, \lambda)$, $(2, -1, 5)$ and $(3, -5, 7\lambda)$ are linearly dependent.

(d) Find the characteristic equation of the matrix

$$\begin{pmatrix} 1 & 2 & 2 \\ 0 & 2 & 1 \\ -1 & 2 & 2 \end{pmatrix}$$ .Also find the eigen values and eigen vectors of this matrix.

(e) Verify the Cayley Hamiltan theorem for the matrix $\begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{pmatrix}$. Also, find its inverse using this theorem.

(f) Diagonalize the matrix $\begin{bmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

Note : Following question no. 5 is for New Syllabus only (TAS - 104/MA - 101(New)).

5. Attempt *any two* parts of the following : **(10x2=20)**

(a) Evaluate by changing the variables, $\iint_R (x+y)^2 \, dx \, dy$ where R is the region bounded by the parallelogram $x+y=0$, $x+y=2$, $3x-2y=0$ and $3x-2y=3$.

(b) Find the volume bounded by the elliptical paraboloids $z = x^2+9y^2$ and $z = 18-x^2-9y^2$.

(c) Using Beta and Gamma functions, evaluate

$$\int_0^1 \left(\frac{x^3}{1-x^3}\right)^{1/2} dx$$

Note : **Following question no.5 is for old syllabus only (MA - 101 (old)).**

5. Attempt *any two* parts of the following : **(10x2=20**

(a) In a binomial distribution the sum and product of the mean and variance of the distribution are 25/3 and 50/3, respectively. Find the distribution.

(b) From the following data which shows the ages X and systolic blood pressure Y of 12 women, find out whether the two variables ages X and blood pressure Y are correlated ?

Ages (X) :	56	42	72	36	63	47	55	49	38	42	68	60
B. P. (Y) :	147	125	160	118	149	128	150	145	115	140	152	155

(c) (i) If Θ is the acute angle between the two regression lines in case of two variables x and y, show that

$$\tan\theta = \frac{1-r^2}{r}.\frac{\sigma_x\sigma_y}{{\sigma_x}^2+{\sigma_y}^2} \text{ where r,}$$

σ_x and σ_y have their usual meanings. Explain the significance of the formula when $r = 0$ and $r = \pm 1$.

(ii) Two variables x and y are correlated by the equation $ax + by + c = 0$. Show that the correlation between them is -1 if signs of a and b are alike and $+1$, if they are different.

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(b) State Gauss divergence theorem. Verify this theorem by evaluating the surface integral as a triple integral

$\int_S \int (x^3 dydz + x^2 ydzdx + x^2 zdxdy)$ where S is the closed surface consisting of the cylinder $x^2 + y^2 = a^2$, $(0 \leq z \leq b)$ and the circular discs $z = 0$ and $z = b$ $(x^2 + y^2 \leq a^2)$.

(c) State the Stokes' theorem. Verify this theorem for $\vec{F}(x, y, z) = xz\hat{i} - y\hat{j} + x^2 y\hat{k}$ where the surface S is the surface of the region bounded by $x = 0, y = 0, z = 0$ $2x + y + 2z = 8$ which is not included on xz-plane.

Attempt *any four* parts of the following : **(5x4=20)**

(a) Find the rank of matrix

$$\begin{bmatrix} 2 & 3 & -2 & 4 \\ 3 & -2 & 1 & 2 \\ 3 & 2 & 3 & 4 \\ -2 & 4 & 0 & 5 \end{bmatrix}$$

(b) Solve the system of equations

$$2x_1 + 3x_2 + x_3 = 9$$

$$x_1 + 2x_2 + 3x_3 = 6$$

$$3x_1 + x_2 + 2x_3 = 8$$

by Gaussian elimination method.

Note

5.