Printed Pages : 7 **EAS-103**

(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 9601 Roll No. | | | | | | | | | | |

B. Tech.

(Only for the candidates admitted/Readmitted in the session 2008-09)

(SEM. I) EXAMINATION, 2008-09

MATHEMATICS - I

Time : 3 Hours] *[Total Marks : 100*

SECTION - A

All parts of this question are **compulsory**. 2×10=20

1 (a) For which value of '*b*' the rank of the matrix

$$A=\begin{bmatrix}1 & 5 & 4\\ 0 & 3 & 2\\ b & 13 & 10\end{bmatrix} \text{ is } 2,\ b = \_\_\_\_\_\_\_\_.$$

(b) Determine the constants a and b such that the curl of vector $\bar{A}=(2xy+3yz)\hat{i} + (x^2+axz-4z^2)\hat{j} - (3xy+byz)\hat{k}$ is zero, $a =$ ________, $b =$ ________.

(c) The n^{th} derivative (y_n) of the function $y=x^2\sin x$ at $x=0$ is ________.

(d) With usual notations, match the items on right hand side with those on left hand side for properties of Max^{m} and minimum. :

(i)	Max^{m}	(p)	$rt - s^2 = 0$
(ii)	Min^{m}	(q)	$rt - s^2 < 0$
(iii)	Saddle point	(r)	$rt - s^2 > 0, r > 0$
(iv)	Failure case	(s)	$rt - s^2 > 0$ and $r < 0$

(e) Match the items on the right hand side with those on left hand side for the following special functions : (Full marks is awarded if all matchings are correct)

(i)	$\beta(p, q)$	(p)	$\Gamma(1/2)$
(ii)	$\frac{\Gamma p \, \Gamma q}{\Gamma p+q}$	(q)	$\int_0^\infty \frac{y^{p-1}}{(1+y)(p+q)} dy$
(iii)	$\sqrt{\pi}$	(r)	$\beta(p, q)$
(iv)	$\frac{\pi}{\sin p\pi}$	(s)	$\Gamma p \, \Gamma 1-p$

Indicate True or False for the following statements :

(f) (i) If $|A| = 0$, then at least one eigen value is zero. (True / False)

(ii) A^{-1} exists iff 0 is an eigen value of A. (True / False)

(iii) If $|A| \neq 0$, then A is known as singular matrix. (True / False)

(iv) Two vectors X and Y is said to be orthogonal Y, $X^T Y = Y^T X \neq 0$. (True / False)

(g) (i) The curve $y^2 = 4ax$ is symmetric about x-axis. (True / False)

(ii) The curve $x^3 + y^3 = 3axy$ is symmetric about the line $y = -x$. (True / False)

(iii) The curve $x^2 + y^2 = a^2$ is symmetric about both the axis x and y. (True / False)

(iv) The curve $x^3 - y^3 = 3axy$ is symmetric about the line $y = x$. (True /False)

Pick the correct answer of the choices given below :

(h) If $\bar{r} = x\hat{i} + y\hat{j} + z\hat{k}$ is position vector, then value of $\nabla(\log r)$ is

(i) $\frac{\bar{r}}{r}$ (ii) $\frac{\bar{r}}{r^2}$

(iii) $-\frac{\bar{r}}{r^3}$ (iv) none of the above.

(i) The Jacobian $\frac{\partial(uv)}{\partial(xy)}$ for the function $u = e^x \sin y$, $v = (x + \log \sin y)$ is

(i) 1 (ii) $\sin x \sin y - xy \cos x \cos y$

(iii) 0 (iv) $\frac{e^x}{x}$

(j) The volume of the solid under the surface $az = x^2 + y^2$ and whose base R is the circle $x^2 + y^2 = a^2$ is given as

(i) $\pi | 2a$ (ii) $\pi a^3 | 2$

(iii) $\frac{4}{3}\pi a^3$ (iv) None of the above.

SECTION - B

Attempt any **three** parts of the following : **10×3=30**

2 (a) If $y = (\sin^{-1} x)^2$ prove that $y_n(0) = 0$ for b odd and $y_n(0) = 2, 2^2, 4^2, 6^2 \ldots (n-2)^2$, $n \neq 2$ for n is even.

(b) Find the dimension of rectangular box of maximum capacity whose surface area is given when (a) box is open at the top (b) box is closed.

(c) Find a matrix P which diagonalizes the matrix $A = \begin{bmatrix} 4 & 1 \\ 2 & 3 \end{bmatrix}$, verify $P^{-1} AP = D$ where D is the diagonal matrix.

(d) Find the area and the mass contained m the first quadrant enclosed by the curve $\left(\frac{x}{a}\right)^{\alpha} + \left(\frac{y}{b}\right)^{\beta} = 1$ where $\alpha > 0$, $\beta > 0$ given that density at any point $p(xy)$ is $k\sqrt{xy}$.

(e) Using the divergence theorem, evaluate the surface integral $\iint_S (yz\,dy\,dz + zx\,dz\,dx + xy\,dy\,dx)$ where $S : x^2 + y^2 + z^2 = 4$.

SECTION - C

Attempt any **two** parts from each question. All questions are **compulsory**.

3 (a) Trace the curve $r^2 = a^2 \cos 2\theta$

(b) If $u = \log\left(\frac{(x^2+y^2)}{(x+y)}\right)$, prove that

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 1$$

(c) If $V = f(2x-3y,\ 3y-4z,\ 4z-2x)$, compute the value of $6V_x + 4V_y + 3V_z$.

4 (a) The temperature 'T' at any point (xyz) in space is $T(xyz) = K\ xyz^2$ where K is constant. Find the highest temperature on the surface of the sphere $x^2 + y^2 + z^2 = a^2$.

(b) Verify the chain rule for Jacobians if $x = u$, $y = u \tan v$, $z = w$.

(c) The time 'T' of a complete oscillation of a simple pendulum of length 'L' is governed by the equation

$T = 2\pi\sqrt{\frac{L}{g}}$, g is constant, find the approximate error in the calculated value of T corresponding to an error of 2% in the value of L.

5 (a) Determine 'b' such that the system of homogeneous equation $2x + y + 2z = 0$; $x + y + 3z = 0$; $4x + 3y + bz = 0$ has (i) Trivial solution (ii) Non-Trivial solution. Find the Non-Trivial solution using matrix method.

(b) Verify Cayley-Hamilton theorem for the matrix

$A = \begin{pmatrix} 1 & 2 \\ 2 & -1 \end{pmatrix}$ and hence find A^{-1}.

(c) Find the eigen value and corresponding eigen vectors of the matrix

$I = \begin{pmatrix} -5 & 2 \\ 2 & -2 \end{pmatrix}$

6 (a) Find the directional derivative of $\nabla(\nabla f)$ at the point $(1, -2, 1)$ in the direction of the normal to the surface $xy^2z = 3x + z^2$ where $f = 2x^3y^2z^4$.

(b) Using Green's theorem, find the area of the region in the first quadrant bounded by the curves

$$y = x,\ y = \frac{1}{x},\ y = \frac{x}{4}.$$

(c) Prove that $\left(y^2 - z^2 + 3yz\right)\hat{i} + \left(3xz + 2xy\right)\hat{j} + \left(3xy - 2xz + 2z\right)\hat{k}$ is both solenoidal and irrotational.

7 (a) Changing the order of integration of

$$\int_0^\infty \int_0^\infty e^{-xy} \sin nx \, dx \, dy$$

Show that $\int_0^\infty \left(\frac{\sin nx}{x}\right) dx = \frac{\pi}{2}$.

(b) Determine the area bounded by the curves $xy = 2$, $4y = x^2$ and $y = 4$.

(c) For a β function, show that

$\beta(p, q) = \beta(p+1, q) + \beta(p, q+1)$.
