(Following Paper ID and Roll No. to be filled in your Answer Book)

| PAPER ID : 9916 | Roll No. | | | | | | | | | | |
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# B.Tech

## (SEM I) ODD SEMESTER THEORY EXAMINATION 2009-10

## MATHEMATICS I

*Time : 3 Hours]* *[Total Marks : 100*

**Note :** *Attempt all questions.*

1 Attempt any **two** parts of the following :

(a) Reduce the matrix :

$$A = \begin{bmatrix} 1 & 1 & -1 & 1 \\ -1 & 1 & -3 & -3 \\ 1 & 0 & 1 & 2 \\ 1 & -1 & 3 & 3 \end{bmatrix}$$

to column echelon form and find its rank.

(b) Verify the Cayley-Hamilton theorem for the matrix :

$$A = \begin{bmatrix} 1 & 0 & -4 \\ 0 & 5 & 4 \\ -4 & 4 & 3 \end{bmatrix}$$

and hence find $A^{-1}$.

(c) Find the eigenvalues and the corresponding eigen vectors of the matrix

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$$

2 Attempt any **two** parts of the following :

(a) If $y = \left(x^2 - 1\right)^n$, prove that

$$\left(x^2 - 1\right) y_{n+2} + 2xy_{n+1} - n(n+1) y_n = 0$$

(b) If $u = x^3 + y^3 + z^3 + 3xyz$, show that

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = 3u$$

(c) If $u = f(r)$ where $r^2 = x^2 + y^2$, prove that

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f''(r) + \frac{1}{r} f'(r)$$

3 Attempt any **two** parts of the following :

(a) If $x = r\sin\theta\cos\phi$, $y = r\sin\theta\sin\phi$, $z = r\cos\theta$, show that

$$\frac{\partial(x, y, z)}{\partial(r, \theta, \phi)} = r^2 \sin\theta$$

(b) Determine the points where the function

$$f(x, y) = x^3 + y^3 - 3xy$$

has a maximum or minimum.

(c) A rectangular box open at the top is to have a given capacity. Find the dimensions of the box requiring least material for its construction.

4 Attempt any **two** parts of the following :

(a) Evaluate

$$\iint_A xy\, dx\, xy$$

where $A$ is the domain bounded by $x-axis$, ordinate $x = 2a$ and the curve $x^2 = 4ay$.

(b) Find the volume bounded by the cylinder $x^2 + y^2 = 4$ and the planes $y + z = 4$ and $z = 0$.

(c) Evaluate :

$$\iiint_R (x + y + z)\, dx\, dy\, dz,$$ where

$$R : 0 \le x \le 1,\ 1 \le y \le 2,\ 2 \le z \le 3$$

5 Attempt any **two** parts of the following :

(a) Find a unit normal vector $\hat{n}$ of the cone of revolution $z^2 = 4\left(x^2 + y^2\right)$ at the point $P : (1, 0, 2)$.

(b) Using Green's theorem evaluate

$$\int_C \left(x^2 + xy\right)dx + \left(x^2 + y^2\right)dy$$

where C is the square formed by the lines $y = \pm 1,\ x = \pm 1$

(c) Verify Stoke's theorem for

$\vec{F} = xy^2\,\hat{i} + y\,\hat{j} + z^2x\,\hat{k}$ for the surface of a rectangular lamina bounded by

$x = 0,\ y = 0,\ x = 1, y = 2,\ z = 0$