(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 9916 Roll No. | | | | | | | | | | |

B.Tech.

(SEM. I) ODD SEMESTER THEORY EXAMINATION 2010-11

MATHEMATICS—I

Time : 3 Hours *Total Marks : 100*

Note : (1) Attempt **all** questions.

(2) All questions carry equal marks.

1. Attempt any **two** parts of the following :— **(10×2=20)**

(a) (i) If $N = \begin{bmatrix} 0 & 1+2i \\ -1+2i & 0 \end{bmatrix}$ is a matrix, then show that $(I - N)(I + N)^{-1}$ is unitary matrix.

(ii) Find the rank of the following matrix by reducing it into Echelon form

$$\begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}.$$

(b) For what values of λ, the equations

$$x + y + z = 1;\ x + 2y + 4z = \lambda,\ x + 4y + 10z = \lambda^2$$

have a solution and solve them completely in each case.

(c) Find the eigen values and the corresponding eigen vectors of the following matrix and hence diagonalize it

$$\begin{bmatrix} 2 & 0 & 1 \\ 0 & 3 & 0 \\ 1 & 0 & 2 \end{bmatrix}$$

2. Attempt any two parts of the following :— (10×2=20)

(a) State Leibnitz theorem.

Find $(y_n)_0$ when $\sin\sqrt{y} = x$.

(b) If $u = (x^2 + y^2 + z^2)^{-1/2}$, show that :

(i) $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} + z\dfrac{\partial u}{\partial z} = -u.$

(ii) $\dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2} + \dfrac{\partial^2 u}{\partial z^2} = 0.$

(c) Expand $f(x, y) = e^x \cos y$ about $\left(1, \dfrac{\pi}{4}\right)$ in Taylor's series upto second degree curves.

3. Attempt any two parts of the following :— (10×2=20)

(a) If $u = x + 2y + z$, $v = x - 2y + 3z$ and $w = 2xy - xz + 4yz - 2z^2$, show that they are not independent. Find the relation between u, v and w.

(b) Find the dimensions of a rectangular box of maximum capacity whose surface is given when the box is open at the top.

(c) (i) The diameter and height of a right circular cylinder are found by measurement to be 8·0 cm and 12·5 cm respectively, with possible errors of 0·05 in each measurement. Find the maximum possible approximate error in the computed volume.

(ii) Trace the curve $r = a \sin 3\theta$.

4. Attempt any **four** parts of the following :— **(5×4=20)**

(a) Prove that $\int_0^{\infty} \sqrt{y}\, e^{-y^2} dy \times \int_0^{\infty} y^{-1/2}\; e^{-y^2}\, dy = \frac{\pi}{2\sqrt{2}}$.

(b) Change the order of integration in the double integral

$$\int_0^{a} \int_{\sqrt{ax-x^2}}^{\sqrt{ax}} f(x, y)\, dx\, dy.$$

(c) Using the transformation $x + y = u$, $y = uv$, evaluate $\iint xy\,(1-x-y)^{1/2}\, dx\, dy$, integration being extended over the area of the triangle bounded by the lines $x = 0$, $y = 0$ and $x + y = 1$.

(d) Show that the area bounded by the curve $x^n + y^n = a^n$ and the coordinate axis in the first quadrant is

$$\frac{a^2\, \{\Gamma(1/n)\}^2}{2n\; \Gamma(2/n)}.$$

(e) Evaluate $\iiint_S x\, dV$, where S is the region bounded by the surfaces $y = x^2$, $y = x + 2$, $4z = x^2 + y^2$ and $z = x + 3$.

(f) Find the mass of a plate which is formed by the coordinate planes and the plane $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$, the variable density being Kxyz.

5. Attempt any **two** parts of the following :— **(10×2=20)**

(a) If $r = |\bar{r}|$, where $\bar{r} = x\hat{i} + y\hat{j} + z\hat{k}$, prove that :

(i) $\nabla f(r) = f'(r)\nabla r$,

(ii) grad r $= \frac{\vec{r}}{r}$,

(iii) $\nabla f(r) \times \vec{r} = 0$

(iv) grad $\left(\frac{1}{r}\right) = -\frac{\vec{r}}{r^3}$

(v) $\nabla \log r = \frac{\vec{r}}{r^2}$

(vi) grad $r^n = n\, r^{n-2}\, \vec{r}$.

(b) State Gauss-Divergence theorem. Verify this theorem for $\vec{F} = (2x - z)\hat{i} + x^2 y\hat{j} - xz^2\hat{k}$ taken over the region bounded by $x = 0, x = 1, y = 0, y = 1$ and $z = 0, z = 1$.

(c) Verify Green's theorem for

$$\int_C [(2x^2 - y^2)dx + (x^2 + y^2)dy],$$

where C is the boundary of the area enclosed by the x-axis and the upper half of the circle $x^2 + y^2 = a^2$.