(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 9601 **Roll No.** | | | | | | | | | | |

B.Tech.

(SEM. I) ODD SEMESTER THEORY EXAMINATION 2010-11

MATHEMATICS—I

Time : 3 Hours *Total Marks : 100*

SECTION–A

1. All parts of this question are compulsory :— **(2×10=20)**

   (a) If $u = f\left(\frac{y}{x}\right)$ then

   $$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \ldots\ldots\ldots\ldots$$

   (b) The curve $x^{2/3} + y^{2/3} = a^{2/3}$

   is symmetrical about ....

   Indicate True or False of the following statements :

   (c) (i) Two functions u and v are functionally dependent if their Jacobian with respect to x and y is zero. (True/False)

   (ii) If $f(x, y) = 1 - x^2y^2$, then stationary point is (0, 0). (True/False)

   (d) (i) The minimum value of $f(x, y) = x^2 + y^2$ is zero. (True/False)

   (ii) If u, v are functions of r, s are themselves function of x, y then $\frac{\partial(u, v)}{\partial(x, y)} = \frac{\partial(u, v)}{\partial(r, s)} \times \frac{\partial(x, y)}{\partial(r, s)}$. (True/False)

   Pick the correct answer of the choices given below :

   (e) The eigen values of $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ are

   (a) 0, 0, 0 (b) 0, 0, 1 (c) 0, 0, 3 (d) 1, 1, 1

(f) The rank of the matrix $\begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$ is

(a) 0 (b) 1 (c) 2 (d) 3

(g) $\dfrac{\beta(m+1, n)}{\beta(m, n)}$ is equal to

(a) $\dfrac{m}{n}$ (b) $\dfrac{m+1}{n}$ (c) $\dfrac{m-1}{n}$ (d) $\dfrac{m}{m+n}$

(h) The value of the integral $\int_0^\infty e^{-x^2}\,dx$ is

(a) $\dfrac{2}{\sqrt{\pi}}$ (b) $\dfrac{\sqrt{\pi}}{2}$ (c) $\dfrac{\pi}{2}$ (d) $\dfrac{2}{\pi}$

Fill up the blanks with the correct answer :

(i) The Gauss divergence theorem relates certain surface integrals to ____________.

(volume integrals/line integrals)

(j) The vector field $\vec{F} = x\hat{i} - y\hat{j}$ is divergence free ____________.

(but not irrotational/and irrotational)

SECTION–B

2. Attempt any **three** parts of the following : **(10×3=30)**

(a) If $y = \sin(a \sin^{-1} x)$. Find $(y_n)_0$.

(b) If u, v, w are the roots of the equation

$(x-a)^3 + (x-b)^3 + (x-c)^3 = 0$, then find $\dfrac{\partial(u, v, w)}{\partial(a, b, c)}$.

(c) Find the eigen values and eigen vectors of the matrix

$$A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}.$$

(d) Change the order of integration in

$$I = \int_0^2 \int_{x^2/4}^{3-x} xy \, dy \, dx$$

and hence evaluate it.

(e) Find the volume enclosed between the two surfaces $Z = 8 - x^2 - y^2$ and $Z = x^2 + 3y^2$.

SECTION–C

Attempt any **two** parts from each question. All questions are compulsory. **(5×2×5=50)**

3. (a) Trace the curve $y^2(a - x) = x^3$.

(b) If $Z = f(x + ct) + \varphi(x - ct)$ show that $\frac{\partial^2 z}{\partial t^2} = c^2 \frac{\partial^2 z}{\partial x^2}$.

(c) Expand $e^{ax} \sin by$ in the powers of x and y as far as terms of third degree.

4. (a) A rectangular box, open at the top, is to have a volume of 32 cubic feet. Determine the dimensions of the box requiring least material for its construction.

(b) If $u_1 = \frac{x_2 x_3}{x_1}$, $u_2 = \frac{x_3 x_1}{x_2}$ and $u_3 = \frac{x_1 x_2}{x_3}$ find the value of $\frac{\partial(u_1, u_2, u_3)}{\partial(x_1, x_2, x_3)}$.

(c) Find the percentage of error in calculating the area of an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, when error of +1% is made in measuring the major and minor axes.

5. (a) Test for consistency and solve the following system of equations

$$2x - y + 3z = 8$$
$$-x + 2y + z = 4$$
$$3x + y - 4z = 0$$

(b) Reduce the following matrix to normal form and hence find its rank :

$$\begin{bmatrix} 5 & 3 & 14 & 4 \\ 0 & 1 & 2 & 1 \\ 1 & -1 & 2 & 0 \end{bmatrix}.$$

(c) Show that the matrix

$$\begin{bmatrix} a+ic & -b+id \\ b+id & a-ic \end{bmatrix}$$

is unitary if and only if $a^2 + b^2 + c^2 + d^2 = 1$.

6. (a) Prove that

$$\int_0^1 \frac{dx}{\sqrt{1+x^4}} = \frac{1}{4\sqrt{2}}\, \beta\left(\frac{1}{4}, \frac{1}{2}\right).$$

(b) Evaluate

$$\iiint x^{\ell-1}\, y^{m-1}\, z^{n-1}\, dx\, dy\, dz\,,$$

where $x > 0, y > 0, z > 0$ under the condition

$$\left(\frac{x}{a}\right)^p + \left(\frac{y}{b}\right)^q + \left(\frac{z}{c}\right)^r \leq 1.$$

(c) Find the area of one loop of the lemniscates

$$r^2 = a^2 \cos 2\theta.$$

7. (a) Find the directional derivative of $\varphi(x, y, z) = xy^2 + yz^3$ at the point $(2, -1, 1)$ in the direction of the normal to the surface $x \log z - y^2 + 4 = 0$ at $(2, -1, 1)$.

(b) If all second order derivatives of φ and $\bar{v}$ are continuous, then show that

(i) $\operatorname{curl}(\operatorname{grad} \varphi) = \bar{0}$

(ii) $\operatorname{div}(\operatorname{curl} \bar{v}) = 0$

(c) Find the work done by the force

$$\bar{f} = (2y+3)\hat{i} + xz\,\hat{j} + (yz - x)\,\hat{k}$$

when it moves a particle from the point $(0, 0, 0)$ to the point $(2, 1, 1)$ along the curve $x = 2t^2$, $y = t$ and $z = t^3$.