(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 9601 Roll No. | | | | | | | | | |

B. Tech.

(SEM. I) THEORY EXAMINATION 2011-12

MATHEMATICS—I

Time : 3 Hours *Total Marks : 100*

SECTION—A

1. All parts of this question are compulsory : (2×10=20)

(a) Find the n^{th} derivative of $x^{n-1} \log x$.

(b) Find the Taylor's series expansion of :

$$f(xy) = x^3 + xy^2 \text{ about point } (2, 1).$$

(c) If $u = e^x \sin y$ and $v = e^x \cos y$, evaluate :

$$\frac{\partial(u, v)}{\partial(x, y)}.$$

(d) Find the minimum value of $x^2 + y^2 + 6x + 12 = 0$.

(e) Find the eigen values of the matrix $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$.

(f) Calculate the inverse of the matrix :

$$\begin{bmatrix} 1 & 2 \\ 5 & 7 \end{bmatrix}.$$

(g) Evaluate $\int_0^1\int_1^2\int_2^3 xyz\,dx\,dy\,dz$.

(h) Evaluate the area enclosed between the parabola $y = x^2$ and the straight line $y = x$.

(i) Find the magnitude of the gradient of the function $f = xyz^3$ at (1, 0, 2).

(j) Write the statement of divergence theorem for a given vector field $\vec{F}$.

SECTION—B

2. Attempt any **three** parts of the following : (10×3=30)

(a) Find the eigen values and eigen vectors of the following matrix :

$$\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -2 & 2 \end{bmatrix}$$

(b) If $y = a\cos(\log x) + b\sin(\log x)$. Find $(y_n)_0$.

(c) The angles of a triangle are calculated from the sides a, b, c. If small changes δa, δb and δc are made in the sides, find δA, δB and δC where Δ is the area of the triangle and A, B, C are angles opposite to sides, a, b, c respectively. Also show that $\delta A + \delta B + \delta C = 0$.

(d) Find the volume bounded by the elliptic paraboloids $z = x^2 + 9y^2$ and $z = 18 - x^2 - 9y^2$.

(e) If $\vec{A} = (x - y)\hat{i} + (x + y)\hat{j}$, evaluate $\oint_C \vec{A}\cdot d\vec{r}$ around the curve C consisting of $y = x^2$ and $y^2 = x$.

SECTION—C

Attempt any **two** parts from each question. All questions are compulsory. **(5×2×5=50)**

3. (a) If $y = \tan^{-1}\left(\dfrac{a+x}{a-x}\right)$, prove that :

$$(a^2 + x^2)\, y_{n+2} + 2(n+1)x\, y_{n+1} + n(n+1)\, y_n = 0.$$

(b) If $u(x, y, z) = \log(\tan x + \tan y + \tan z)$, prove that

$$\sin 2x \frac{\partial u}{\partial x} + \sin 2y \frac{\partial u}{\partial y} + \sin 2z \frac{\partial u}{\partial z} = 2.$$

(c) Trace the curve :

$$r^2 = a^2 \cos 2\theta.$$

4. (a) If $y_1 = \dfrac{x_2 x_3}{x_1}$, $y_2 = \dfrac{x_3 x_1}{x_2}$ and $y_3 = \dfrac{x_1 x_2}{x_3}$, find the value of $\dfrac{\partial (y_1, y_2, y_3)}{\partial (x_1, x_2, x_3)}$.

(b) Find the extreme values of :

$$f(x, y) = x^3 + y^3 - 3\,axy$$

(c) If the base radius and height of a cone are measured as 4 cm and 8 cm. with a possible error of 0.04 and 0.08 inches respectively, calculate the percentage (%) error in calculating volume of the cone.

5. (a) Define curl of a vector. Prove the following vector identity :

$$\mathrm{Div}(\vec{u} \times \vec{v}) = \mathrm{Curl}\,\vec{u} \cdot \vec{v} - \mathrm{Curl}\,\vec{v} \cdot \vec{u}.$$

(b) If $r = (x^2 + y^2 + z^2)^{1/2}$, evaluate $\nabla^2 (\log r)$.

(c) Find the surface area of the plane $x + 2y + 2z = 12$ cut off by $x = 0$, $y = 0$ and $x^2 + y^2 = 16$.

6. (a) Express the Hermitian Matrix :

$$A = \begin{bmatrix} 1 & -i & 1+i \\ i & 0 & 2-3i \\ 1-i & 2+3i & 2 \end{bmatrix}$$

as P + iQ where P is a real symmetric and Q is a real skew symmetric matrix.

(b) Using elementary row transformations, find the inverse of the following matrix :

$$A = \begin{bmatrix} 2 & 3 & 4 \\ 4 & 3 & 1 \\ 1 & 2 & 4 \end{bmatrix}$$

(c) State and verify Cayley-Hamilton theorem for the following matrix :

$$A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

7. (a) Find the mass of a plate which is formed by the co-ordinate planes and the plane $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$, the density is given by $\rho = k\,xyz$.

(b) Using Beta and Gamma functions, evaluate $\int_0^\infty \frac{dx}{1+x^4}$.

(c) Evaluate the integral $\int_0^a \int_0^{\sqrt{a^2 - y^2}} \left(x^2 + y^2\right) dx\, dy$ by changing into polar coordinates.