Printed Pages—4 **AS101**

(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 1106 **Roll No.** | | | | | | | | | | |

B.Tech.

(SEM. I) ODD SEMESTER THEORY EXAMINATION 2013-14

MATHEMATICS - I

Time : 3 Hours *Total Marks : 100*

Note :– Attempt **all** Sections.

SECTION–A

1. **All** parts of this question are compulsory : **(10×2=20)**

   (a) Find the n^{th} derivative of $y = x^2 \sin x$.

   (b) Find the stationary points of :
   $f(x, y) = 5x^2 + 10y^2 + 12xy - 4x - 6y + 1$

   (c) Find all the asymptotes of the curve :
   $xy^2 = 4a^2 (2a - x)$.

   (d) Find the envelope of the family of straight lines $y = mx + \frac{a}{m}$, where m is a parameter.

   (e) Compute $\Gamma\left(\frac{-5}{2}\right)$.

   (f) Evaluate eigen values of $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$.

   (g) Prove that $\vec{A} = (6xy + z^3)\hat{i} + (3x^2 - z)\hat{j} + (3xz^2 - y)\hat{k}$ is irrotational.

(h) Evaluate $\int_0^a \int_0^x xy \, dy \, dx$.

(i) Find a unit vector normal to the surface :
$x^3 + y^3 + 3xyz = 3$ at the point (1, 2, –1).

(j) Determine the rank of the matrix :

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

SECTION–B

2. Attempt any **three** parts of the following : **(3×10=30)**

(a) If $y = (\sin^{-1}x)^2$, prove that :
$(1 - x^2)\, y_{n+2} - (2n + 1)\, xy_{n+1} - n^2 y_n = 0$ and calculate $y_n(0)$.

(b) Divide 24 into three parts such that the continued product of the first, square of the second and the cube of the third may be maximum.

(c) Find the volume contained in the solid region in the first Octant of the ellipsoid :

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

(d) Verify Green's theorem in plane for :

$$\oint_C (x^2 - 2xy)\, dx + (x^2 y + 3)\, dy$$ where C is the boundary of the region defined by $y^2 = 8x$ and $x = 2$.

(e) Diagonalise the unitary matrix $A = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & i \\ -i & -1 \end{bmatrix}$.

SECTION–C

Note :– Attempt any **two** parts from each question. All questions are compulsory : **(5×2×5=50)**

3. (a) Expand $e^{2x} \sin x$ in ascending powers of x upto x^5.

(b) If x = tan (log y), prove that :

$$(1 + x^2)\, y_{n+2} + [2 (n + 1) x - 1]\, y_{n+1} + n(n + 1)\, y_n = 0.$$

(c) Find the centre of circle of curvature for :

$xy(x + y) = 2$ at (1, 1).

4. (a) If $J = \frac{\partial(u,v)}{\partial(x,y)}$ and $J^* = \frac{\partial(x,y)}{\partial(u,v)}$ then show that $J.J^* = 1$.

(b) Show that :

$$xU_x + yU_y + zU_z = -2 \cot u.$$

where $u = \cos^{-1}\left(\frac{x^3 + y^3 + z^3}{ax + by + cz}\right)$

(c) Find approximate value of :

$$[(3.82)^2 + 2(2.1)^3]^{1/5}$$

5. (a) Evaluate $I = \int_0^1 \left(\frac{x}{1-x^3}\right)^{1/2} dx$.

(b) Evaluate $\iint_R (x+y)^2\, dx\,dy$ where R is region bounded by the parallelogram

$x + y = 0,\ x + y = 2,\ 3x - 2y = 0,\ 3x - 2y = 3.$

(c) Compute the area bounded by the lemniscate $r^2 = a^2 \cos 2\theta$.

6. (a) If $\vec{A} = (x-y)\hat{i} + (x+y)\hat{j}$, evaluate $\oint_C \vec{A} \cdot \overline{dr}$ around the curve C consisting of $y = x^2$ and $y^2 = x$.

(b) Find the directional derivative of :

$\phi = (x^2 + y^2 + z^2)^{-1/2}$ at the point (3, 1, 2) in the direction of the vector $yz\hat{i} + zx\hat{j} + xy\hat{k}$.

(c) Evaluate $\oint_C \vec{F} \cdot \overrightarrow{dr}$ by Stoke's Theorem, where :

$\vec{F} = y^2\hat{i} + x^2\hat{j} - (x+z)\hat{k}$ and C is the boundary of triangle with vertices at (0, 0, 0), (1, 0, 0) and (1, 1, 0).

7. (a) Test the consistency and solve the following system of equations :

$$2x - y + 3z = 8$$

$$-x + 2y + z = 4$$

$$3x + y - 4z = 0$$

(b) If $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{bmatrix}$, find the inverse of A using Cayley-Hamilton Theorem.

(c) If $A = \begin{bmatrix} 2 & 3+2i & -4 \\ 3-2i & 5 & 6i \\ -4 & -6i & 3 \end{bmatrix}$

Then show that A is Hermitian and iA is Skew-Hermitian.