(Following Paper ID and Roll No. to be filled in your Answer Books)

Paper ID : 2012439 **Roll No.** | | | | | | | | | | |

B.TECH.

Regular Theory Examination (Odd Sem - I),2016-17

ENGINEERING MATHEMATICS-I

Time : 3 Hours *Max. Marks : 70*

Note : The question paper contains three sections - A, B & C. Read the instructions carefully in each section.

SECTION - A

Attempt all questions of this section. Each part carries 2 marks.

1. a) For what value of 'x', the eigen values of the given matrix A are real

$$A = \begin{bmatrix} 10 & 5+i & 4 \\ x & 20 & 2 \\ 4 & 2 & -10 \end{bmatrix}$$ **(2)**

b) For the given matrix $A = \begin{bmatrix} -5 & -3 \\ 2 & 0 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ prove that $A^3 = 19A + 30I$. **(2)**

c) Find the maximum value of the function $f(xyz) = (z - 2x^2 - 2y^2)$ where $3xy - z + 7 = 0$. **(2)**

d) If the volume of an object expressed in spherical coordinates as following :

$V = \int_0^{2\pi}\int_0^{\pi}\int_0^{1} r^2 \sin\phi \, dr \, d\phi \, d\theta$ Evaluate the value of V.**(2)**

e) Find the condition for the contour on $x - y$ plane where the partial derivative of $(x^2 + y^2)$ with respect to y is equal to the partial derivative of $(6y + 4x)$ with respect to x. **(2)**

f) The parabolic arc $y = \sqrt{x}$, $1 \le x \le 2$ is resolved around x - axis. Find the volume of solid of revolution. **(2)**

g) For the scalar field $u = \frac{x^2}{2} + \frac{y^2}{3}$, Find the magnitude of gradient at the point (1, 3). **(2)**

SECTION - B

Attempt any three parts of the following. Each part carries 7 marks.

2. a) i) Express $2A^5-3A^4+A^2-4I$ as a linear polynomial in A where $A=\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$. **(3)**

ii) Reduce the matrix $P=\begin{bmatrix} 1 & 2 & -2 \\ 1 & 2 & 1 \\ -1 & -1 & 0 \end{bmatrix}$ to diagonal form. **(4)**

b) i) If $u=\sin^{-1}\left(\dfrac{x^{1/4}+y^{1/4}}{x^{1/6}+y^{1/6}}\right)$ then evaluate the value of $\left(x^2\dfrac{\partial^2 u}{\partial x^2}+2xy\dfrac{\partial^2 u}{\partial x\partial y}+y^2\dfrac{\partial^2 u}{\partial y^2}\right)$ **(3)**

ii) Trace the curve $x = a(\theta - \sin\theta)$, $y = a(1 - \cos\theta)$. **(4)**

c) i) Find the relation between u, v, w for the values $u = x + 2y + z$; $v = x - 2y + 3z$; $w = 2xy - zx + 4yz - 2z^2$. **(3)**

ii) Divide a number into three parts such that the product of first, square of the second and cube of third is maximum. **(4)**

d) i) Change the order of integration for $I = \int_0^1 \int_{x^2}^{2-x} xy\, dx dy$ and hence evaluate the same. **(3)**

ii) Evaluate the triple integral $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} (xyz)\, dx\, dy\, dz$. **(4)**

e) i) If $\vec{F}=\left(x^2+y^2\right)\hat{i}-2xy\,\hat{j}$, then evaluate the value of $\oint \vec{F}.d\bar{r}$. **(3)**

ii) Find the directional derivative of $\left(\frac{1}{r^2}\right)$ in the direction of $\vec{r}$ where $\vec{r}=\hat{i}x+\hat{j}y+\hat{k}z$. **(4)**

SECTION - C

Attempt all questions of this section, selecting any two parts from each question. All questions carry equal marks. (5×7=35)

3. a) If $I_n=\frac{d^n}{dx^n}\left(x^n\log x\right)$, show that $I_n=nI_{n-1}+\lfloor n-1$.

b) If $e^{-z/\left(x^2-y^2\right)}=x-y$ then show that $y\frac{\partial z}{\partial x}+x\frac{\partial z}{\partial y}=x^2-y^2$.

c) If $w=\sqrt{x^2+y^2+z^2}$ & $x=\cos v$, $y=u\sin v$, $z=uv$,

then prove that $\left[u\frac{\partial w}{\partial u}-v\frac{\partial w}{\partial v}\right]=\frac{u}{\sqrt{1+v^2}}$.

4. a) If $x=v^2+w^2$, $y=w^2+u^2$, $z=u^2+v^2$ then show that $\frac{\partial(xyz)}{\partial(uvw)}\cdot\frac{\partial(uvw)}{\partial(xyz)}=1$.

b) Express the function $f(xy)=x^2+3y^2-9x-9y+26$ as Taylor's Series expansion about the point (1, 2).

c) Find the percentage error in measuring the volume of a rectangular box when the error of 1% is made in measuring the each side.

5. a) If $A=\begin{bmatrix}-3 & 2\\ -1 & 0\end{bmatrix}$ then evaluate the value of the expression $(A+5I+2A^{-1})$.

b) Find the eigen value of the matrix $\begin{bmatrix}4 & 2\\ 2 & 4\end{bmatrix}$ corresponding to the eigen vector $\begin{bmatrix}101\\ 101\end{bmatrix}$.

c) Show that $A = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & 1 & 1 \\ 1 & w & w^2 \\ 1 & w^2 & w \end{bmatrix}$ is a unitary matrix, where w is complex cube root of unity.

6. a) Changing the order of integration in the double integral $I = \int_0^8 \int_{x/4}^2 f(xy)\,dy\,dx$ leads to the value $I = \int_r^s \int_p^q f(xy)\,dx\,dy$. What is the value of q?

b) Evaluate $\iiint x^2 yz \, dx\, dy\, dz$ through out the volume bonded by planes $x=0, y=0, z=0$ & $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$.

c) For the Gamma function, show that

$$\frac{\Gamma\left(\frac{1}{3}\right)\Gamma\left(\frac{5}{6}\right)}{\Gamma\left(\frac{2}{3}\right)} = (2)^{1/3}\sqrt{\pi}.$$

7. a) Verify Stokes theorem $\vec{F} = (2y+z,\ x-z,\ y-x)$ taken over the triangle ABC cut from the plane $x+y+z=1$ by the coordinate planes.

b) Verify Gauss Divergence theorem for $\int_c \left[\left(x^3 - yz\right)\hat{i} - 2x^2 y\hat{j} + 2\hat{k}\right]\hat{n}\,ds$ where S denotes the surface of cube bounded by the planes $x = 0, x = a;$ $y = 0, y = a; z = 0, z = a.$

c) If $\vec{A} = \left(xz^2\hat{i} + 2y\hat{j} - 3xz\hat{k}\right)$ and $\vec{B} = \left(3xz\hat{i} + 2yz\hat{j} - z^2\hat{k}\right)$ Find the value of $\left[\vec{A}\times\left(\nabla\times\vec{B}\right)\right]$ & $\left[\left(\vec{A}\times\nabla\right)\times\vec{B}\right]$.

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