B.Tech.
(SEM-I) THEORY EXAMINATION 2018-19
MATHEMATICS-I

Time: 3 Hours   *Total Marks: 100*

Note: Attempt all Sections. If require any missing data; then choose suitably.

SECTION A

1.   Attempt all questions.

Q no.	Question	Marks	CO
a.	Find the rank of the matrix $\begin{bmatrix} 2 & 2 & 2 \\ 2 & 2 & 2 \\ 2 & 2 & 2 \end{bmatrix}$.	2	1
b.	Find the stationary point of $f(x,y) = x^3 + y^3 + 3axy, a > 0$	2	3
c.	If $x = r\cos\theta,\ y = r\sin\theta,\ z = z$ then find $\frac{\partial(r,\theta,z)}{\partial(x,y,z)}$	2	3
d.	Define del ∇ operator and gradient.	2	5
e.	If $\phi = 3x^2 y - y^3 z^2$, find grad ϕ at point (2, 0, -2).	2	5
.f.	Evaluate $\int\limits_0^1 \int\limits_0^{x^2} e^{\frac{y}{x}}\, dx dy$.	2	4
g.	If the eigen values of matrix A are 1, 1, 1, then find the eigen values of $A^2 + 2A + 3I$.	2	1
h.	Define Rolle's Theorem	2	2
i.	If $u = x^3 y^2 \sin^{-1}(y/x)$, then find $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$.	2	3
j.	In RI = E and possible error in E and I are 20 % and 10 % respectively, then find the error in R.	2	3
k.	State the Taylor's Theorem for two variables.	2	3

SECTION B

2.   Attempt any three of the following:

Q no.	Question	Marks	CO
a.	Using Cayley- Hamilton theorem find the inverse of the matrix A= $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{bmatrix}$.	10	1

Also express the polynomial B= $A^8-11A^7-4A^6+A^5+A^4-11A^3-3A^2+2A+I$
as a quadratic polynomial in A and hence find B.

b. If $y = \text{Sin}(m \sin^{-1}x)$, prove that : $(1 - x^2)\, y_{n+2} - (2n + 1)x\, y_{n+1} - (n^2 - m^2)y_n = 0$ and find y_n at $x = 0$. 10 2

c. If u, v, w are the roots of the equation $(x - a)^3 + (x - b)^3 + (x - c)^3 = 0$, then find $\dfrac{\partial(u, v, w)}{\partial(a, b, c)}$. 10 3

d. Evaluate $\displaystyle\int_0^\infty \int_0^\infty e^{-(x^2 + y^2)}\,dxdy$ by changing to polar coordinates. 10 4

Hence show that $\displaystyle\int_0^\infty e^{-x^2}\,dx = \dfrac{\sqrt{\pi}}{2}$.

e. Verify the divergence theorem for $\vec{F} = \left(x^3 - yz\right)\hat{i} + \left(y^3 - zx\right)j + \left(z^3 - xy\right)\hat{k}$, taken over the cube bounded by planes $x = 0, y = 0, z = 0, x = 1, y = 1, z = 1$. 10 5

SECTION C

3. Attempt any *one* part of the following:

Q no.	Question	Marks	CO
a.	Find inverse employing elementary transformation $A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$	10	1
b.	Reduce the matrix A to its normal form when $A = \begin{bmatrix} 1 & 2 & -1 & 4 \\ 2 & 4 & 3 & 4 \\ 1 & 2 & 3 & 4 \\ -1 & -2 & 6 & -7 \end{bmatrix}$. Hence find the rank of A.	10	1

4. Attempt any *one* part of the following:

Q no.	Question	Marks	CO
a.	If $\sin^{-1} y = 2\log(x + 1)$ show that $(x + 1)^2 y_{n+2} + (2n + 1)(x + 1)y_{n+1} + (n^2 + 4)y_n = 0$	10	2
b.	Verify Lagrange's Mean value Theorem for the function $f(x) = x^3$ in $[-2, 2]$	10	2

5. Attempt any *one* part of the following:

Q no.	Question	Marks	CO
a.	Find the maximum or minimum distance of the point $(1, 2, -1)$ from the sphere $x^2 + y^2 + z^2 = 24$.	10	3
b.	If $u = \cos^{-1}\left(\dfrac{x + y}{\sqrt{x} + \sqrt{y}}\right)$ then show that $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} + \dfrac{1}{2}\cot u = 0$	10	3

6. **Attempt any *one* part of the following:**

Q no.	Question	Marks	CO
a.	Change the order of integration and then evaluate: $\int_{0}^{2} \int_{\frac{x^2}{4}}^{3-x} x\, y \ dy\, dx$.	10	4
b.	Calculate the volume of the solid bounded by the surface x=0, y=0, x+y+z=1 & z=0.	10	4

7. **Attempt any-*one* part of the following:**

Q no.	Question	Marks	CO
a.	Prove that $\left(y^2 - z^2 + 3yz - 2x\right)\hat{i} + \left(3xz + 2xy\right)\hat{j} + \left(3xy - 2xz + 2z\right)\hat{k}$ is both Solenoidal and Irrotational.	10	5
b.	Find the directional derivative of $\Phi = 5x^2y - 5y^2z + \frac{5}{2}z^2x$ at the point P(1, 1, 1) in the direction of the line $\frac{x-1}{2} = \frac{y-3}{-2} = \frac{z}{1}$.	10	5