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TAS-204

(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID: 9929

Roll No.

B. Tech.

(SEM. II) EXAMINATION, 2007-08 **MATHEMATICS - II**

Time: 3 Hours]

[Total Marks: 100

- **Note**: (1) Attempt all questions.
 - (2) All questions carry equal marks.
 - (3) In case of numerical problems assume data wherever not provided.
 - (4) Be precise in your answer.
- 1 Attempt any four parts of the following: $5\times4=20$

(a) Solve
$$\left(y + \sqrt{x^2 + y^2}\right) dx - x dy = 0$$
,

godes
$$y(1) = 0$$
. The same and the slow of the pro-

(b) Solve the following differential equation:

$$(\cos x - x \cos y)dy - (\sin y + y \sin x)dx = 0$$

(c) Find the solution of following differential equation:

$$(D^2 - 4D - 5)y = e^{2x} + 3\cos(4x + 3)$$

where
$$D = \frac{d}{dx}$$
.

(d) Solve the following simultaneous differential equations:

$$\frac{dx}{dt} + 5x - 2y = t$$

$$\frac{dy}{dt} + 2x + y = 0$$

Also show that x = y = 0 when t = 0.

- (e) Solve by method of variation of parameters $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = e^{-x} \log x.$
- (f) An inductance (L) of 2.0 H and a resistance (R) of 20 Ω are connected in series with an e.m.f. E volt. If the current (i) is zero, when t=0, find the current (i) at the end of 0.01 second if E=100 V, using the following

$$L\frac{di}{dt} + iR = E$$

differential equation:

$$\int_0^\infty e^{-t} \, \frac{\sin^2 t}{t} \, dt \, .$$

(b) State second shifting theorem for Laplace transform and hence find the Laplace transform of the following function

$$F(t) = \begin{cases} e^{t-a}, & t > a \\ 0, & t < a \end{cases}$$

(c) Using convolution theorem, find the inverse

Laplace transform of the following
$$\frac{s}{(s^2 + a^2)^3}$$
.

(d) Solve the following simultaneous differential equations by Laplace transform

$$\frac{dx}{dt} + 4\frac{dy}{dt} - y = 0$$

$$\frac{dx}{dt} + 2y = e^{-t}$$

with condition x(0) = y(0) = 0.

Laplace transform
$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 5y = e^{-x} \sin x$$

where
$$y(0) = 0$$
, $\left(\frac{dy}{dx}\right)_{x=0} = 1$.

- (f) Using unit step function, find the Laplace transform of:
 - (i) $(t-1)^2 \cdot u(t-1)$
 - (ii) $\sin t \cdot u(t-\pi)$.
- 3 Attempt any two parts of the following: 10×2=20
 - (a) Solve the following differential equation in series:

$$2x^{2}\frac{d^{2}y}{dx^{2}} - x\frac{dy}{dx} + (x-5)y = 0.$$

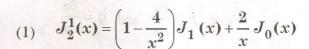
(b) Show that

(1)
$$\int_{-1}^{+1} \left[P_n(x) \right]^2 dx = \frac{2}{n+1}$$

(2)
$$P_{2n}(0) = \frac{(-1)^n (2n)!}{2^{2n} (n!)^2}$$

where $P_n(x)$ is the Legendre polynomial of degree n.

(c) Show that



(2)
$$J_0^2 + 2(J_1^2 + J_2^2 + J_3^2 +) = 1$$

where $J_n(x)$ is the Bessel's polynomial of degree n and dash denote the differentiation.

- 4 Attempt any two parts of the following: 10×2=20
 - (a) Obtain a Fourier series to represent $f(x) = x^2$ in the interval $(0, 2\pi)$ and hence deduce that

$$\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \dots = \frac{\pi^2}{12}.$$

(b) Examine whether the function $f(x) = x \sin x$ is even or odd. Hence expand it in the form of Fourier series in the interval $(-\pi, \pi)$.

(c) Solve the following partial differential equations:

(1)
$$x(y^2+z)p-y(x^2+z)q=z(x^2-y^2)$$

(2)
$$\frac{\partial^2 z}{\partial x^2} - 2 \frac{\partial^2 z}{\partial x \partial y} = \sin x \cdot \cos 2y$$

where
$$p = \frac{\partial z}{\partial x}$$
 and $q = \frac{\partial z}{\partial y}$.

- 10×2 5 Attempt any two parts of the following:
 - Solve the following equation by the method of separation of variables:

$$\frac{\partial u}{\partial x} = 4 \frac{\partial u}{\partial y}$$
 where $u(0, y) = 8 e^{-3y}$.

(b) Solve the following Laplace equation:

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

in a rectangle with
$$u(0, y) = 0$$
, $u(a, y) = 0$,

$$u(x, b) = 0$$
 and $u(x, 0) = f(x)$ along

x-axis.

(c) Assuming the resistance of wire (R) and conductance to ground (C_T) are negligible, find the voltage v(x, t) and current i(x, t) in a transmission line of length l, t seconds after the ends are suddenly grounded. The initial conditions are $v(x, 0) = v_0 \sin\left(\frac{\pi x}{l}\right)$ and $i(x, 0) = i_0$.