Printed Pages : 8 **EAS – 203**

(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 9610 Roll No. ____________

B. Tech.

(SEM. II) EXAMINATION, 2008-09

MATHEMATICS - II

Time : 3 Hours] *[Total Marks : 100*

SECTION - A

Note : Attempt all questions. All questions carry equal marks. **2×10=20**

Fill up the appropriate answers in the space provided :

1 (a) The differential equation

$$\left(\frac{d^3y}{dx^3}\right)^4 - 6x^2\left(\frac{dy}{dx}\right)^2 + e^x = \sin xy \text{ is of}$$

__________ order and __________ degree.

(b) A particular solution of the differential equation

$\frac{dy}{dt} = \frac{1}{2}(y^2 - 1)$ that satisfies the initial condition

$y(0) = 2$ is __________.

(c) The general solution of the differential equation

$\frac{d^5y}{dx^5} - \frac{d^3y}{dx^3} = 0$ is given by __________.

Pick up the Correct Answer from the following :

(d) The Rodrigues formula for Legendre Polynomial $P_n(x)$ is given by

(i) $P_n(x) = \frac{1}{\underline{|n}\, 2^n} \frac{d^n}{dx^n}(x^2-1)^n$

(ii) $P_n(x) = \frac{\underline{|n}}{2^n} \frac{dn}{dx^n}(x^2-1)^{n-1}$

(iii) $P_n(x) = \frac{\underline{|n}}{2^{n-1}} \frac{d^n}{dx^n}(x^2-1)^{n-1}$

(iv) $P_n(x) = \frac{1}{\underline{|n}\, 2^n}(x^2-1)^n$

(e) The Laplace transform of the function

$f(t) = \begin{cases} 1, & 0 \le t < 2 \\ -1, & 2 \le t < 4, \end{cases}$ $f(t+4) = f(t)$ is given as

(i) $\frac{1-e^{-2s}}{s(1+e^{-2s})}$

(ii) $\frac{1+e^{-2s}}{s(1+e^{-2s})}$

(iii) 0

(iv) $\frac{s+1}{s-1}$

(f) The inverse Laplace transform of $\log\left(\frac{s+1}{s-1}\right)$ is given by

(i) $\frac{2}{t}\cosh t$

(ii) $\frac{2}{t}\sin ht$

(iii) $2t\cos t$

(iv) $2t\sin t$

Indicate **True / False** for the statements made therein -

(g) (i) A function $f(x)$ is even if $f(-x)=-f(x)$ (True / False)

(ii) A function $f(x)$ is odd if $f(-x)=f(x)$. (True / False)

(iii) Most functions are neither even nor odd. (True / False)

(iv) A function $f(x)$ can always be expressed as an arithmetic mean of an even and odd function as

$$f(x)=\frac{1}{2}[f(x)+f(-x)]+\frac{1}{2}[f(x)-f(-x)].$$

(True / False)

(h) (i) With usual symbols, the PDE

$u_{xx}+u^2u_{yy}=f(xy)$ is non-linear in 'u' and is of second order. (True / False)

(ii) The small transverse vibrations of a string are governed by one dimensional heat equation $y_t = a^2 y_{xx}$. (True / False)

(iii) Two dimensional steady state heat flow is given by Laplace's equation $u_t = a^2(u_{xx} + u_{tt})$. (True / False)

(iv) The PDE of all sphere whose centre lie on z-axis and given by equations $x^2 + y^2 + (z-a)^2 = b^2$, a and b being constants, are governed by $x z_y - y z_x = 0$. (True / False)

(i) Applying the method of separation of variables techniques, the solution to the P.D.E. $3u_x + 2u_y = 0$ is _________, where $u_x = \frac{\partial u}{\partial x}$, $u_y = \frac{\partial u}{\partial y}$.

Match the column for the items of the Left side to that of right side :

(j) A second order P.D.E. in the function *'u'* of two independent variables x, y given with usual symbols $Au_{xx} + Bu_{xy} + Cu_{yy} + F(u) = 0$, then

(i)	Hyperbolic	(a)	$B^2 - 4AC = 0$
(ii)	Parabolic	(b)	$B^2 - 4AC < 0$
(iii)	Elliptic	(c)	$B^2 - 4AC > 0$
(iv)	Not classified	(d)	$A = B = C = 0$

SECTION - B

Note : Attempt any **three** questions from this section. All questions carry equal marks. **3×10=30**

2 (a) Apply the method of variation of parameters to solve the ordinary differential equations

$$\frac{d^2y}{dx^2}+y=\tan x,\ 0<x<\pi/2.$$

(b) Show that Bessel's function $J_n(x)$ is an even function when n is even and is odd function when n is odd. Express $J_6(x)$ in terms of $J_0(x)$ and $J_1(x)$.

(c) Use convolution theorem to find the inverse of the function $\dfrac{1}{(s^2+a^2)^2}$.

(d) Obtain the Fourier series of $f(x)=\left(\dfrac{\pi-x}{2}\right)$ in the interval $(0,\ 2\pi)$ and hence deduce

$$\frac{\pi}{4}=1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\ldots.$$

(e) A string of Length $'L'$ is stretched and fastened to two fixed points. Find the solution of the wave equation $y_{tt}=a^2y_{xx}$ when initial displacement is $y(x,\ 0)=f(x)=b\sin\left(\dfrac{\pi x}{L}\right)$, where symbols have usual meaning.

SECTION - C

Note : Attempt all questions from this section, selecting any **two** parts from each question. All questions carry **equal** marks. **[5×2]×5=50**

3 (a) The equations of motion of a particle are given by

$$\frac{dx}{dt}+wy=0,\ \frac{dy}{dt}-wx=0.$$ Find the path of the particle and show that it is a circle.

(b) Integrate the differential equation

$$\frac{d^2x}{dt^2}+2x\frac{dx}{dt}+w^2x=a\cos pt$$ and give the physical interpretation of the complete solution. Also deduce that the solution takes the form

$$x=\frac{a\cos(pt-\theta)}{\sqrt{(w^2-p^2)^2+4k^2p^2}}\left(\tan\theta=\frac{2kp}{w^2-p^2}\right)$$ As $t\to\infty$

(c) Find the complete solution of the differential equation $$\frac{d^2y}{dx^2}-2\frac{dy}{dx}+y=x\,e^x\cos x.$$

4 (a) For the Bessel's function, prove that

$$J_{\frac{1}{2}}(x)=\left(\sqrt{\frac{2}{\pi x}}\right)\cdot\sin x.$$

(b) Estimate the recurrence relation for Legendre's polynomials -

$$\boxed{x\,P_n'(x) = n\,P_n(x) + P_{n-1}'(x)}$$

(c) Express the polynomial $f(x) = 4x^3 - 2x^2 - 3x + 8$ in terms of Legendre polynomials.

5 (a) Using Laplace transformation, show that

$$\int_0^\infty \frac{\sin t}{t}\,dt = \frac{\pi}{2}.$$

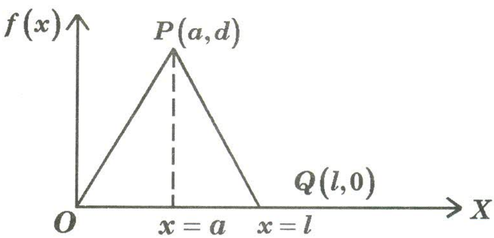
(b) A particle moves in a line so that its displacement x from a fixed point **0** at any time t, is given by

$$\frac{d^2x}{dt^2} + 4\frac{dx}{dt} + 5x = 80\sin 5t$$. Using Laplace transform, find its displacement at any time t if initially particle is at rest at $x = 0$.

(c) Find the Laplace transform of the function

$$f(t) = \begin{cases} t-1, & 1 < t < 2 \\ 3-t, & 2 < t < 3 \end{cases}$$

6 (a) Find the half period sine series for $f(x)$ given in the range $(0, l)$ by the graph OPQ as shown in figure.



(b) Find the Fourier series expansion for

$$f(x) = x + \frac{x^2}{4}, \quad -\pi \le x \le \pi.$$

(c) Solve the P.D.E.

$$\frac{\partial^2 z}{\partial x^2} - 2\frac{\partial^2 z}{\partial x\,\partial y} + \frac{\partial^2 z}{\partial y^2} = \sin x.$$

7 (a) Find the temperature in a bar of length 2 whose end are kept at zero and lateral surface is insulated if the initial temperature is $\sin\frac{nx}{2} + 3\sin 5\pi\frac{x}{2}$.

(b) Apply the method of separation of variables to solve $\frac{\partial^2 z}{\partial x^2} - 2\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = 0$.

(c) Solve the P.D.E. by separation of variables method, $u_{xx} = u_y + 2u$, $u(0, y) = 0$, $\frac{\partial}{\partial x}u(0, y) = 1 + e^{-3y}$.