Printed Pages : 8 **EAS-203**

(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 9610 Roll No.

B. Tech. (Second Semester) Theory Examination, 2010-11

MATHEMATICS–II

Time : 3 Hours] *[Total Marks : 100*

Note : Attempt the questions from each Section as indicated. The symbols have their usual meaning.

Section-A

Attempt *all* questions of this Section. Each question carries equal marks. $2\times10=20$

1. (i) In the RC-circuit where $C=0.01$ F; $R=20$ ohms, $E_0=10$ V. The current $I(t)$, assuming that capacitor is completely uncharged, is given as $I(t)=$ ______

   (ii) The differential equation for which $xy=ae^{x}+be^{-x}+x^2$ is the solution, is given as ______.

(iii) If $\frac{d}{dx}(J_0) = nJ_1$, then value of n is ______.

(iv) The roots of Indicial equations for the power series solution of the differential equation $2x^2y'' + xy' + (x^2 - 3)y = 0$ are ______.

(v) The value of Inverse Laplace Transform $L^{-1}\left\{\frac{1}{s^{3/2}}\right\}$ is given as :

(a) $\sqrt{\frac{t}{\pi}}$

(b) $2\sqrt{\frac{t}{\pi}}$

(c) $t^{3/2}$

(d) $\sqrt{t}$.

(vi) If Laplace Transform of $L\{f(t)\} = \frac{e^{-1/s}}{s}$, then $L\{e^{-t}f(3t)\}$ is :

(a) $\frac{e^{-1/(s+1)}}{(s+1)}$

(b) $\frac{e^{-3/(s+1)}}{(s+1)}$

(c) $\frac{e^{-3/s}}{s}$

(d) $\frac{e^{-3/s}}{(s-1)}$.

(vii) State True or False for the following statements :

(a) x and $\sin x$ are even functions. (True/False)

(b) The function $f(x) = \cos 5x$ is a periodic function with period 2π. (True/False)

(c) The product of an even function and an odd function is an odd function. (True/False)

(d) The graph of an odd function is symmetrical about y-axis. (True/False)

(viii) The partial differential equation

$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = \frac{1}{c^2}\frac{\partial u}{\partial t}$$

is :

(a) Elliptic

(b) Parabolic

(c) Hyperbolic

(d) None of these.

(ix) Match the following :

(a) One-dimensional heat equation (p) $\frac{\partial^2 u}{\partial x^2} = \frac{1}{h}\frac{\partial u}{\partial t}$

(b) One-dimensional wave equation (q) $\frac{\partial^2 u}{\partial x^2} = -\frac{\partial^2 u}{\partial y^2}$

(c) Two-dimensional Laplace equation (r) $\frac{\partial^2 u}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2}$

(d) Two-dimensional heat equation (s) $\frac{\partial^2 u}{\partial x^2} = \frac{1}{c^2}\frac{\partial u}{\partial t} - \frac{\partial^2 u}{\partial y^2}$

(x) The general solution of partial differential equation $p - q = \log(x + y)$ is given as _____.

Section-B

Attempt any *three* parts of the following. 10×3=30

2. (a) Find the complete solution of the differential equation $(D-2)^3 y = 17 e^{2x}$, where symbols have their usual meanings.

(b) Using power series method, obtain the solution for $x^2 y'' + x(x-1) y' + (1-x) y = 0$, about $x=0$.

(c) Using the Laplace Transform, solve the differential equation $y'' + 2y' + 5y = e^{-t} \sin t$, $y(0) = 0,\ y'(0) = 1$.

(d) Obtain the Fourier series expansion of $f(x) = \left(\frac{\pi - x}{2}\right)$ in $0 < x < 2$.

(e) Find the temperature distribution in a bar of length 2 whose ends are kept at zero temperature and lateral surface insulated if the initial temperature is :

$$\sin\left(\frac{\pi x}{2}\right) + 3\sin\left(\frac{5x\pi}{2}\right).$$

Section-C

All questions in this Section are compulsory with the choices indicated in each question. Symbols have their usual meaning. 10×5=50

3. Attempt any two parts of the following :

(a) Solve by method of variation of parameter $(D^2 - 1)y = 2(1 - e^{-2x})^{-1/2}$.

(b) Solve the system of simultaneous differential equations :

$$\frac{dx}{dt}+x-2y=0,\ \frac{dy}{dt}+x+4y=0,\quad x(0)=y(0)=1.$$

(c) Find the steady state solution in RLC circuit equation consisting of inductance L=0.05 H, resistance R=05 ohms and a condensor of capacitance 4×10^{-4} farad, if $Q=I=0$ when $t=0$ and there is an alternating emf of 200 cos 100 t. Find $Q(t)$ and $I(t)$.

4. Attempt any two of the following :

(a) Express the polynomial $f(x)=4x^3-2x^2-3x+8$ in terms of Legendre polynomials.

(b) Show that :

$$J_1''(x)=-J_1(x)+\frac{1}{x}J_2(x).$$

(c) Evaluate :

$$\int x^2 J_1(x)dx.$$

5. Attempt any two of the following :

(a) Use convolution theorem to find the inverse of :

$$\left[\frac{16}{(s-2)(s+2)^2}\right].$$

(b) Find the Laplace Transform of following periodic function :

$$f(t)=\begin{cases}\sin\omega t, & 0<t<\frac{\pi}{\omega}\\ 0, & \frac{\pi}{\omega}<t<\frac{2\pi}{\omega}.\end{cases}$$

(c) Show that :

$$\int_{t=0}^{\infty}\int_{u=0}^{t}\frac{e^{-t}\sin u}{u}\,du\,dt=\frac{\pi}{4}$$

6. Attempt any two of the following :

(a) Find half range sine series of the function $f(x)=x-x^2$ in $0<x<1$.

(b) Find the Fourier series of :

$$f(x)=\begin{cases}0, & \text{when } -\pi\le x\le 0\\ x^2, & \text{when } 0\le x\le\pi\end{cases}.$$

(c) Find the general solution of partial differential equation :

$$(y^2 + z^2)\,p - xyq + zx = 0.$$

7. Attempt any one of the following :

(a) In a telephone of wire of length l, a steady voltage distribution of 20 volts at the source end and 12 volts at the terminal end is maintained. At time $t=0$, the terminal is grounded. Assuming $L=0$, $G=0$, determine the voltage and current where symbols have their usual meanings.

(b) The temperature distribution in a bar of length π which is perfectly insulated at ends $x=0$ and $x=\pi$, governed by the partial differential equation $\frac{\partial u}{\partial t} = k^2 \frac{\partial^2 u}{\partial x^2}$. Assuming the initial temperature as $u(x, 0) = f(x) = \cos \epsilon\, x$, find the temperature distribution at any instant of time, k and ϵ being constants.