(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 9929 **Roll No.** | | | | | | | | | | |

B. Tech.

(SEM. II) THEORY EXAMINATION 2010-11

MATHEMATICS–II

Time : 3 Hours *Total Marks : 100*

Note : Attempt **all** questions. All questions carry equal marks.

1. Attempt any **four** parts of the following : **(5×4=20)**

(a) Solve the equation $x\frac{dy}{dx}-\frac{1}{2}y=x+1$ and prove that the only solution for which x and y can attain the value unity is given by $y=2x+\sqrt{x}-2$.

(b) Solve $\frac{d^2y}{dx^2}+5\frac{dy}{dx}-6y=\sin 3x+\cos 2x$.

(c) Solve $(x+1)^2\frac{d^2y}{dx^2}+(x+1)\frac{dy}{dx}=(2x+3)(2x+4)$.

(d) Solve the simultaneous equations :

$$\frac{dx}{dt}-y=e^t,\quad \frac{dy}{dt}+x=\sin t$$

given $x(0)=1$, $y(0)=0$.

(e) Solve the following differential equation by reducing into normal form

$$\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + (x^2+2)y = e^{\frac{1}{2}(x^2+2x)}.$$

(f) An RL circuit has an e.m.f. given (in volts) by 4 sin t, a resistance of 100 ohms, an inductance of 4 henry and no initial current. Find the current at any time t.

2. Attempt any **two** parts of the following : **(10×2=20)**

(a) Using Frobenius method, obtain a series solution about $x = 0$ for the differential equation :

$$2x^2y'' - xy' + (1 - x^2)y = 0.$$

(b) (i) Prove that $\int_{-1}^{1} P_m(x)\ P_n(x)\, dx = \frac{2}{2n+1}$, $m = n$.

(ii) Prove that $n\ P_n(x) = x\ P_n'(x) - P_{n-1}'(x)$.

(c) (i) Prove that $J_{5/2}(x) = \sqrt{\frac{2}{\pi x}}\left[\left(\frac{3-x^2}{x^2}\right)\sin x - \frac{3\cos x}{x}\right]$

(ii) Prove that $J_n''(x) = \frac{1}{4}\left[J_{n-2}(x) - 2J_n(x) + J_{n+2}(x)\right]$.

3. Attempt any **four** parts of the following : **(5×4=20)**

(a) State first shifting property for Laplace transform. Hence find the Laplace Transform of

$$f(t) = t^2\, e^t \sin 4t.$$

(b) Use Laplace transform to evaluate :

$$\int_0^\infty e^{-t} \frac{\sin^2 t}{t} \, dt$$

(c) Express the following functions in terms of unit step function and hence find its Laplace transform :

$$f(t) = \begin{cases} \sin t, & 0 < t < \pi \\ \sin 2t, & \pi < t \end{cases}$$

(d) Find the function whose Laplace transform is :

$$F(s) = \frac{s-1}{s^2(s-7)}$$

(e) Solve the following integral equation :

$$y(t) = 1 - \sinh t + \int_0^t (1+u)\, y(t-u)\, du.$$

(f) Using Laplace transform, solve :

$$\frac{d^2x}{dt^2} + 9x = \sin 2t, \quad x(0) = 1, \quad x'(0) = 0.$$

4. Attempt any **two** parts of the following : **(10×2=20)**

(a) Find the Fourier series of the periodic function f(x) with period 2π defined as follows :

$$f(x) = \begin{cases} 0, & -\pi < x \le 0 \\ x, & 0 \le x \le \pi \end{cases}.$$

Hence prove that $\sum_{n=1}^{\infty} \frac{1}{(2n-1)^2} = \frac{\pi^2}{8}$

(b) Given the function $f(x) = x, 0 < x < 1$, find :

(i) Fourier cosine series for $f(x)$,

(ii) Fourier sine series for $f(x)$.

(c) Solve $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2} - 3\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = xy + e^{x+2y}$.

5. Attempt any **two** parts of the following : **(10×2=20)**

(a) Use the method of separation of variables to obtain the general solution of

$$c^2 \frac{\partial^2 z}{\partial x^2} = \frac{\partial u}{\partial t}$$

that tends to zero as $t \to \infty$ for all x.

(b) Find the steady state temperature distribution in a semicircular plate of radius a, insulated on both faces with its curved boundary kept at a constant temperature U_0 and its bounding diameter is kept at zero temperature.

(c) A string is stretched and fastened to two points distance L apart. Find the displacement of the string at any point x and at any time t if the motion is started by displacing the string in the form $y = a \sin^3 \frac{\pi x}{L}$ from which it is released at time $t = 0$.