Printed Pages : 4 **AS-201**

(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 1212 **Roll No.** | | | | | | | | | | |

B. Tech.

(Semester-II) Even Semester Theory Examination, 2012-13

ENGINEERING MATHEMATICS-II

Time : 3 Hours] *[Total Marks : 100*

Note : Attempt questions from each Section as per instructions. The symbols have their usual meaning.

Section-A

Attempt *all* parts of this question. Each part carries 2 marks. 2×10=20

1. (a) Verify for exactness the differential equation :

$$y\exp(xy).dx+[x\exp(xy)+2y]dy=0.$$

(b) State the criterion for linearly independent solutions of the homogeneous linear nth order differential equation.

(c) Explain ordinary and singular points of a differential equation of the form :

$$a_2(x)y''+a_1(x)y'+a_0(x)y=0.$$

(d) Prove that :

$$J_0'(x)=-J_1(x).$$

(e) If Laplace transform of $f(t)$ is $F(s)$, then show that Laplace transform of $[e^{at}f(t)]$ is $F(s-a)$, where a is any real number.

(f) Find :

$$L^{-1}\left[\frac{1}{s^2-3s+3}\right].$$

(g) Find the partial differential equation which is satisfied by the relation $z=c_1xy+c_2$, where c_1 and c_2 are constants.

(h) Give two examples of non-linear partial differential equation of the first order.

(i) Write telegraph equations.

(j) Characterize the following partial differential equation into elliptic, parabolic and hyperbolic equations :

$$a\frac{\partial^2 z}{\partial x^2}+2h\frac{\partial^2 z}{\partial x \partial y}+b\frac{\partial^2 z}{\partial y^2}+2f\frac{\partial z}{\partial x}+2g\frac{\partial z}{\partial y}+cz=f(x, y),$$

where a, b, c, h, f, g are constants.

Section-B

Attempt any *three* parts of this question. Each part carries 10 marks. 10×3=30

2. (a) A light elastic string of length l is hung by one end. To the other end are tied successively particles of masses m_1 and m_2 and they produced statical extensions l_1 and l_2. If T_1 and T_2 are the periods of small oscillations corresponding to the two masses, show that :

$$\frac{T_1^2-T_2^2}{l_1-l_2}=\frac{4\pi^2}{g}.$$

(b) Find the series solution of the differential equation $xy''+y'+x^2y=0$.

(c) Find the solution of the differential equation :

$$\frac{d^2y}{dt^2}-2\frac{dy}{dt}+y=e^t$$

with the initial conditions $y(t)=0$ and $\frac{dy}{dt}=1$ when $t=0$ using Laplace transform.

(d) Solve the partial differential equation $(p^2+q^2)y=qz$.

(e) Solve the one-dimensional heat equation given that $u(0, t)=0$, $u(l, t)=0$ and $u(x, 0)=x, 0<x<l$, where l is the length of the bar.

Section-C

Attempt any *two* parts from each questions of this Section. Each question carries 10 marks. 5×2×5=50

3. (a) Solve the differential equation :

$$x^2\frac{d^2y}{dx^2}+4x\frac{dy}{dx}+2y=e^x.$$

(b) Solve by changing the independent variable :

$$x\frac{d^2y}{dx^2}+(4x^2-1)\frac{dy}{dx}+4x^3y=2x^3.$$

(c) Solve the system :

$$\dot{x}(t)=y,\ \dot{y}(t)=-x, \qquad x(0)=0,\ y(0)=0.$$

4. (a) Prove that :

$$J_{-n}(x)=(-1)^n J_n(x).$$

(b) Prove that :

$$P_n(x)=\frac{1}{2^n.n!}\frac{d^n}{dx^n}(x^2-1)^n.$$

(c) Prove that :

$$\int_{-1}^{1}P_n^2(x)dx=\frac{2}{2n+1}$$

5. (a) Using Laplace transform, evaluate $\int_0^\infty \frac{e^{-t}\sin t}{t}dt.$

(b) Find the inverse Laplace transform of $\cot^{-1}\left(\frac{s+3}{2}\right).$

(c) State convolution theorem and hence evaluate $L^{-1}\left\{\frac{s}{(s^2+1)(s^2+4)}\right\}.$

6. (a) Obtain Fourier series of the function :

$$f(x)=\begin{cases} x\ , & -\pi<x<0 \\ -x\ , & 0<x<\pi \end{cases}.$$

(b) Find the half range cosine series of the function $f(x)=x\sin x,\ 0<x<\pi.$

(c) Solve $(D^2-2DD'+D'^2)\,z=\sin x.$

7. (a) Solve by the method of separation of variables :

$$\frac{\partial^2u}{\partial x^2}=\frac{\partial u}{\partial y}+2u,$$

$$u(0,\ y)=0 \text{ and } \frac{\partial u}{\partial x}(0,\ y)=1+e^{-3y}.$$

(b) A string is stretched and fastened to two points distance l apart. Find the displacement $y(x, t)$ at any point at a distance x from one end at time t, given that :

$$y(x, 0) = A\sin\left(\frac{2\pi x}{l}\right).$$

(c) Using the transformations $u = x - ct$ and $v = x + ct$, transform the equation :

$$\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$$

to the form $\frac{\partial^2 y}{\partial u \partial v} = 0$. Hence show that $y(x, t) = f_1(x - ct) + f_2(x + ct)$.