(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 199201 **Roll No.** | | | | | | | | | | |

B.Tech.

(SEM.II) THEORY EXAMINATION 2013-14

MATHEMATICS–II

Time : 3 Hours *Total Marks : 100*

Note :– Attempt **all** Sections.

1. Attempt all parts of this question. Each part carries 2 marks. **(10×2=20)**

SECTION–A

(a) Find the general solution of $(2D + 1)^2 y = 0$.

(b) Form a differential equation if its general solution is $y = Ae^{x} + Be^{-x}$.

(c) If $L\{F(t)\} = \dfrac{e^{-1/s}}{s}$, find the $L\{e^{-t} F(3t)\}$.

(d) Find Laplace transform of $\sin 2t\, u(t-\pi)$.

(e) Express $2x^2 + x + 3$ in terms of Legendre polynomials.

(f) If $J_{1/2}(x) = \sqrt{\dfrac{m}{\pi x}} \sin kx$, then find m and k.

(g) If $F(x) = \begin{cases} -x, & -\pi < x < 0 \\ x & 0 < x < \pi \end{cases}$ find $F(0)$.

(h) Solve $(D^2 + DD')z = 0$.

(i) Classify the following partial differential equation $(f_{xx} + 2f_{xy} + 4f_{yy}) = 0$.

(j) Write down the telegraph equations.

SECTION–B

Note :- Attempt any **three** parts of this question. Each part carries equal marks. **(3×10=30)**

2. (a) Solve the following differential equations by the method of variation of parameters

$y_2 - 3y_1 + 2y = e^{2x} + x^2$

(b) Solve in series $(x + x^2 + x^3)\dfrac{d^2y}{dx^2} + 3x^2\dfrac{dy}{dx} - 2y = 0$.

(c) Solve the equation by Laplace transform

$(D^3 - D^2 - D + 1)\,y = 8\,te^{-t}$; $y(0) = 0,\ y'(0) = 1,\ y''(0) = 0$

(d) Obtain Fourier series for the function

$$f(x) = \begin{cases} 1 + \dfrac{2x}{\pi}, & -\pi \le x \le 0 \\ 1 - \dfrac{2x}{\pi}, & 0 \le x \le \pi \end{cases}$$

(e) Find the deflection of the vibrating string of unit length whose end points are fixed if the initial velocity is zero and the initial deflection is given by

$$u(x, 0) = \begin{cases} 1, & 0 \le x \le \dfrac{1}{2} \\ -1, & \dfrac{1}{2} \le x \le 1 \end{cases}$$

SECTION–C

Note :- Attempt any two parts from each question of this section. Each part carries equal marks. **(2×5×5=50)**

3. (a) Solve the system of simultaneous differential equations :

$\dfrac{dx}{dt} = -4(x + y),\ \dfrac{dx}{dt} + 4\dfrac{dy}{dt} = -4y$ with conditions $x(0) = 1,\ y(0) = 0$

(b) Solve the differential equation : $\dfrac{d^2y}{dx^2} + y = \cosh 2x + x^3$

(c) Solve the differential equation by changing the independent variable :

$$x^6 \frac{d^2y}{dx^2} + 3x^5 \frac{dy}{dx} + a^2 y = \frac{1}{x^2}$$

4. (a) Solve the differential equation in series : $y'' + xy' + (x^2 + 2)y = 0$

(b) Prove that :

$$\int_{-1}^{1} (x^2 - 1) P_{n+1} P_n' dx = \frac{2n(n+1)}{(2n+1)(2n+3)}$$

(c) Prove that :

$$J_4(x) = \left(\frac{48}{x^3} - \frac{8}{x}\right) J_1(x) + \left(1 - \frac{24}{x^2}\right) J_0(x)$$

5. (a) Find the Laplace transform of the following functions :

(i) $te^{-t}\cosh t$

(ii) $\int_0^t e^t \frac{\sin t}{t} dt$

(b) Find $L^{-1}\left[\log\left(\frac{s^2+4s+5}{s^2+2s+5}\right)\right]$

(c) Use convolution theorem to find $L^{-1}\left[\frac{16}{(s-2)(s+2)^2}\right]$.

6. (a) If $f(x) = \begin{cases} x, & 0 < x < \dfrac{\pi}{2} \\ \pi - x, & \dfrac{\pi}{2} < x < \pi \end{cases}$

Hence show that

$$f(x) = \frac{4}{\pi}\left[\sin x - \frac{\sin 3x}{3^2} + \frac{\sin 5x}{5^2} - \dots\dots\right]$$

(b) Solve $(y + zx)\,p - (x + yz)\,q = x^2 - y^2$

(c) Solve $\dfrac{\partial^3 z}{\partial x^3} - 7\dfrac{\partial^3 z}{\partial x \partial y^2} - 6\dfrac{\partial^3 z}{\partial y^3} = \sin(x + 2y)$

7. (a) Use method of separation of variables to solve the equation

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial y} + 2u$$

(b) Solve $u_t = a^2 u_{xx}$ under the conditions $u(0, t) = 0$, $u(l, t) = 0$ $(t > 0)$ and initial condition $u(x,0) = x(10-x)$, l being the length of the bar.

(c) A square plate is bounded by the lines $x = 0$, $y = 0$, $x = 20$, $y = 20$. Its faces are insulated. The temperature along the upper horizontal edge is given by $u(x, 20) = x(20 - x)$ when $0 < x < 20$ while other three edges are kept at 0°C. Find the steady state temperature in the plate.