

(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID: 9967

Roll No.

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B.Tech.

THIRD SEMESTER EXAMINATION, 2006 - 07

COMPUTER BASED NUMERICAL AND STATISTICAL TECHNIQUES

Time : 3 Hours

Total Marks : 100

Note : (i) Answer ALL questions.

(ii) All questions carry equal marks.

(iii) In case of numerical problems assume data wherever not provided.

(iv) Be precise in your answer.

1. Attempt any four parts of the following : (5x4=20)

(a) Find the relative error, absolute error and percentage error, if $\frac{2}{3}$ is approximated to 0.6667.

(b) The function $f(x) = \tan^{-1}x$ can be expanded as

$$\tan^{-1}x = x - \frac{x^3}{3} + \frac{x^5}{5} - \dots + (-1)^{n-1} \frac{x^{2n-1}}{2n-1} + \dots$$

find n such that the series determine $\tan^{-1}x$ correct to eight significant digits.

(c) Using Regula-Falsi method, compute the smallest positive root of the equation $xe^x - 2 = 0$, correct upto four decimal places.

- (d) Use Newton's Raphson method to find the smallest positive root of the equation $\tan x = x$.
- (e) Compute the rate of convergence of Newton-Raphson method.
- (f) Find the number of real and complex roots of the polynomial equation $x^4 - 4x^3 + 3x^2 + 4x - 4 = 0$ using Sturm sequence.

2. Attempt *any four* parts of the following : (5x4=20)

- (a) Compute $f(27)$ from the following data using Lagrange's interpolation formula.

$x :$	14	17	31	35
$f(x) :$	68.7	64.0	44.0	39.1

- (b) Find the polynomial of degree four which takes the following values :

$x :$	2	4	6	8	10
$y :$	0	0	1	0	0

- (c) Obtain the Newton's divided difference interpolating polynomial and hence find $f(6)$.

$x :$	3	7	9	10
$f(x) :$	168	120	72	63

- (d) Find the value of $\int_0^{\pi/2} \sqrt{1 - 0.162 \sin^2 x} dx$ using Simpson's one-third rule taking 6 sub-intervals.
- (e) The velocity ' v ' of a particle at distance ' s ' from a point on its linear path is given in the following table :

s (m) :	0	2.5	5.0	7.5	10.0	12.5	15.0	17.5	20.0
v (m/sec) :	16	19	21	22	20	17	13	11	9

Estimate the time taken by the particle to traverse the distance of 20 meters, using Boole's rule.

- (f) Compute $\int_0^{\pi/2} \sin x \, dx$, using Simpson's three eighth rule of integration, taking $h = \frac{\pi}{18}$.

3. Attempt *any two* parts of the following : (10x)

- (a) Using Bessel's formula, compute the value of $f(1.95)$ from the following data :

$x :$	1.7	1.8	1.9	2.0	2.1	2.2	2.3
$f(x) :$	2.979	3.144	3.283	3.391	3.463	3.997	4.121

- (b) If $y(10) = 35.3$, $y(15) = 32.4$, $y(20) = 29.5$, $y(25) = 26.1$, $y(30) = 23.2$ and $y(35) = 20.5$, find $y(12)$ using Newton's forward as well as backward interpolation formula. Also explain why the difference (if any) in the result occurs.
- (c) Find the values of $f''(5)$ and $f'''(0.5)$ from the following table :

$x :$	0	1	2	3	4	5
$f(x) :$	4930	5026	5122	5217	5312	5407

4. Attempt *any two* parts of the following : (10x2)

- (a) Find the value of $y(1.1)$, using Runge-Kutta method of fourth order, given that

$$\frac{dy}{dx} = y^2 + xy, \quad y(1) = 1.0, \quad \text{take } h = 0.05$$

- (b) Given that $\frac{dy}{dx} = 1 + y^2$; $y(0.6) = 0.684$

$$y(0.4) = 0.4228, \quad y(0.2) = 0.2027, \quad y(0) = 0. \quad \text{Find}$$

$y(-0.2)$, using Milne's predictor-corrector method.

- (c) Find $y(0.1)$, using improved Euler's method and then $y(0.2)$ by using modified Euler's method, given that

$$\frac{dy}{dx} = \log(x + y), \quad y(0) = 1.0$$

5. Attempt *any two* parts of the following : (10x2=20)

- (a) Obtain cubic spline for every subinterval, given in the tabular form :

$x :$	0	1	2	3
$f(x) :$	1	2	33	244

with the end conditions $M_0 = 0 = M_3$.

- (b) Two variables x and y have zero means, the same variance σ^2 and zero correlation, show that :

$$u = (x \cos \alpha + y \sin \alpha) \text{ and}$$

$$v = (x \cos \alpha - y \sin \alpha)$$

have the same variance σ^2 and zero correlation.

- (c) The data below given the number of defective bearing in samples of size 150. Construct np-chart for these data. If any points lie outside the control limits, assume that assignable cause can be found and determine the revised control limits :

Sample no. :	1	2	3	4	5	6	7	8	9	10
No. of defectives :	12	7	5	4	1	5	9	0	15	6

Sample no. :	11	12	13	14	15	16	17	18	19	20
No. of defectives :	7	4	1	3	6	8	10	5	2	7

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