(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 1064 Roll No. | | | | | | | | | |

B.Tech.

THIRD SEMESTER EXAMINATION, 2006-07

DISCRETE STRUCTURE

Time : 3 Hours *Total Marks : 100*

Note : (i) *Attempt ALL questions.*

(ii) *All questions carry equal marks.*

(iii) *Be precise in your answer.*

1. Attempt *any four* parts of the following : (5x4=20)

(a) (i) Show that for any two sets A and B $A - (A \cap B) = A - B$.

(ii) Give the power set of the set given below :

$A = \{a, \{b\}\}$

(b) (i) Let R be a binary relation defined as $R = \{<a, b> \in R^2 \mid a - b \leq 3\}$ determine whether R is reflexive, symmetric, anti symmetric and transitive.

(ii) How many distinct binary relations are there on the finite set A ?

(c) Let $X = \{1, 2, \ldots, 7\}$ and

$R = \{<x, y> \mid x - y \text{ is divisible by } 3\}$

show that R is an equivalence relation.

(c) Define a group. Let $S = \{0, 1, 2, 3, 4, 5, 6, 7\}$ and * denote "multiplication modulo 8 i.e. $x*y = (xy) \bmod 8$.

Write three distinct groups (G, *) where $G \subset S$ and G has two elements.

(d) What do you mean by group homomorphism and group isomorphism ? Explain with example.

(e) If $(R, +, \bullet)$ is a ring with unity, then show that, for all $a \in R$.

(i) $(-1).a = -a$

(ii) $(-1).(-1) = 1$

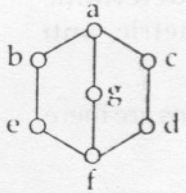
(f) Find the elements and the multiplication table of the symmetric groups S_3.

3. Attempt *any four* parts of the following : (5x4=20)

(a) Define Poset. Give an example of a set X such that $(P(x), \subseteq)$ is a totally ordered set.

(b) Let A be a given finite set and P (A) its power set. Let $\subseteq$ be the inclusion relation on the elements of P (A). Draw the Hasse diagrams of $(P(A), \subseteq)$ for $A = \{a, b, c\}$.

(c) In the lattice defined by the Hasse given by the following figure :



How many complements does the elements '*e*' have ? Give all.

(d) List all possible functions from $X = \{a, b, c\}$ to $y = \{0, 1\}$ and indicate in each case whether the function is one to one, is onto and is one to one onto.

(e) (i) Define an equivalence class generated by the elements of a set on a given equivalence relation.

(ii) Let F_x be the set of all one to one onto mapping from X onto X, where $X = \{1, 2, 3\}$. Find all the elements of F_x and find the inverse of each element.

(f) State and prove Pigeon hole principle.

2. Attempt *any four* parts of the following : **(5x4=20)**

(a) Let (A, *) be a semigroup, further more for every a and b in A, if $a \neq b$, then $a*b \neq b*a$.

(i) Show that for every a in A

$a*a = a$

(ii) Show that for every a, b in A

$a*b*a = a$

(iii) Show that for every a, b, c in A

$a*b*c = a*c$

(b) Let G_1 and G_2 be sub group of a group G.

(i) Show that $G_1 \cap G_2$ is also a subgroup of G.

(ii) Is $G_1 \cup G_2$ always a subgroup of G ?

(d) Define a boolean function. For any x and y in a boolean algebra show that $\overline{x \vee y} = \bar{x} \wedge \bar{y}$.

(e) Write the following Boolean expressions in an equivalent product of sums canonical form in three variables x_1, x_2 and x_3.

(i) $x_1 * x_2$

(ii) $x_1 \oplus x_2$

(f) Define following terms :

(i) Rooted tree

(ii) Binary tree

(iii) Binary search tree

4. Attempt *any two* parts of the following : **(10x2=20)**

(a) (i) What is difference between conditional and biconditional statements ? Explain with example.

(ii) Make a truth table for : $(P \to Q) \wedge (P \to R)$.

(b) Show that the truth values of the following formulas are independent of their components.

(i) $(P \wedge (P \to Q)) \to Q$

(ii) $(P \to Q) \rightleftarrows (\neg P \vee Q)$

(iii) $((P \to Q) \wedge (Q \to R)) \to (P \to R)$

(c) Show that given formula is a tautology

$((P \vee Q) \wedge \neg(\neg P \wedge (\neg Q \vee \neg R))) \vee (\neg P \wedge \neg Q) \vee (\neg P \wedge \neg R)$.

5. Attempt *any two* parts of the following : **(10x2=20)**

(a) Solve the following recurrence relations :

(i) $a_{n+1} - 1.5\, a_n = 0,\ n \geq 0$

(ii) $a_n = 5a_{n-1} + 6a_{n-2},\ n \geq 2,\ a_0 = a_1 = 3$

(b) Describe the 1-isomorphism and 2-isomorphism of the graph with example.

(c) Write short notes on any two of the following :

(i) Complete bipartite graph

(ii) Hamiltonian paths and circuit

(iii) Chromatic number of a graph

(iv) Eular graphs

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