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Printed Pages : 8 ECS303

(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 0111 Roll No. | | | | | | | | | | |

B.Tech

(SEM III) ODD SEMESTER THEORY EXAMINATION 2009-10
DISCRETE MATHEMATICAL STRUCTURES

Time : 3 Hours] *[Total Marks : 100*

Note : *(i) Answer all questions.*
(ii) All questions carry equal marks.

1 Attempt any **four** parts : **5×4=20**

(a) Let A be a set with 10 distinct elements. Determine the following :

(i) Number of different binary relations on A

(ii) Number of different symmetric binary relations on A

(b) Suppose S and T are two sets and f is a functions from S to T. Let R_1 be an equivalence relation on T. Let R_2 be a binary relation on S such that $(x, y) \in R_2$ if and only if $(f(x), f(y)) \in R_1$. Show that R_2 is also an equivalence relation.

(c) Prove that union of two countably infinite set is countably infinite.

(d) Composition of functions is commutative. Prove the statement or give counter example.

(e) Prove that for any integer $n > 1$,

$$\frac{1}{\sqrt{1}}+\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{3}}+\dots\dots\dots+\frac{1}{\sqrt{n}}>\sqrt{n}.$$

(f) What do you understand by asymptotic behaviour of a numeric function ? Explain Big-Oh and Big-Omega notation.

2 Attempt any **two** parts : 10×2=2

(a) (i) Define group. Prove that if every element of a group G is its own inverse then G is abelian group.

(ii) Prove that $(\mathbb{Z}_p, X_p)$ is a group where the set $\mathbb{Z}_p$ is the set of all non-zero residue classes modulo p, p is a positive prime number and X_P denotes the multiplication of residue classes modulo p.

(b) (i) Prove the following or give counter example :

If $(H_1, *)$ and $(H_2, *)$ are both subgroups of the group $(G, *)$ then $(H_1 \cup H_2, *)$ is also a subgroup of $(G, *)$.

(ii) State and prove Lagrange theorem.

(c) (i) Define and explain the following with suitable example :

(a) Cyclic group

(b) Zero divisor of a ring

(c) Order of an element of a group

(d) Field.

(ii) If G is a group of order n then order of any element $a \in G$ is a factor of n. Prove.

Answer any **two** parts : **10×2=20**

(a) (i) Define a relation R on the set $\mathbb{Z}$ of all integers by mR_n if and only if $m^2 = n^2$. Determine whether R is a partial order or not ?

(b) (i) Prove the following or give counter example :

If $(H_1, *)$ and $(H_2, *)$ are both subgroups of the group $(G, *)$ then $(H_1 \cup H_2, *)$ is also a subgroup of $(G, *)$.

(ii) State and prove Lagrange theorem.

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Answer any **two** parts : **10×2=20**

(a) (i) Define a relation R on the set $\mathbb{Z}$ of all integers by mR_n if and only if $m^2 = n^2$. Determine whether R is a partial order or not ?

(ii) Let $(A, \leq)$ and $(B, \leq)$ be two posets. Prove that $(A \times B, \leq)$ is a poset, where $(a, b) \leq (c, d)$ if and only if $a \leq c,\ b \leq d$.

(iii) Draw the Hasse diagram of $(A, \leq)$ where

$$A = \{3,\ 4,\ 12,\ 24,\ 48,\ 72\}$$

and the relation $\leq$ be such that $a \leq b$ if a divides b.

(b) (i) Define distributive lattice and complemented lattice. Prove that in a distributive lattice, if an element has a complement then this complement is unique.

(ii) Let

$$E(x_1,\ x_2,\ x_3) = \overline{\overline{(x_1 \vee x_2)} \vee (\bar{x}_1 \wedge x_3)}$$

be a boolean expression over the two valued boolean algebra. Write $E(x_1,\ x_2,\ x_3)$ in disjunctive normal form.

(c) (i) Let a, b, c be elements in a lattice $(A, \leq)$. Show that if $a \leq b$ then
$a \vee (b \wedge c) \leq b \wedge (a \vee c)$.

(ii) Simplify boolean function F given by
$$F(A, B, C, D) = \sum (0, 2, 7, 8, 10, 15)$$
using Karnaugh map.

4 Answer any **two** parts :

(a) (i) Given that the value of $p \rightarrow q$ is false, determine the value of $(\bar{p} \vee \bar{q}) \rightarrow q$.

(ii) Find a formula A that uses the variable p, q and r such that A is a contradiction.

(iii) Write an equivalent formula for
$p \wedge (q \leftrightarrow r) \vee (r \leftrightarrow p)$
which neither contains biconditional nor conditional connectives.

(iv) The contrapositive of a statement S is given as "If $x < 2$ then $x + 4 < 6$". write the statement S and its converse.

(b) (i) Prove that $(p \vee q) \Rightarrow (p \wedge q)$ is logically equivalent to $p \Leftrightarrow q$.

(ii) Translate the following sentences in quantified expressions of predicate logic.

(a) all students need financial aid.

(b) Some students need financial aid.

(c) (i) Show that following are not equivalent :

(a) $\forall x (P(x) \rightarrow Q(x))$ and $\forall x P(x) \rightarrow \forall x Q(x)$

(b) $\forall x \; \exists y \; P(x, y)$ and $\exists y \; \forall x \; P(x, y)$.

(ii) Show that $r \rightarrow \sim q, \; r \vee s, \; s \rightarrow \sim q, \; p \rightarrow q \Leftrightarrow \sim p$ are inconsistent.

5 Answer any **two** parts :

(a) (i) Find the simple expression for the generating function of following discrete numeric function

$$1, \frac{2}{3}, \frac{3}{9}, \frac{4}{27}, \ldots\ldots, \frac{(r+1)}{3^r}, \ldots\ldots$$

(ii) Solve the recurrence relation

$$a_r - 6\,a_{r-1} + 8\,a_{r-2} = r \cdot 4^r$$

given $a_0 = 8,\ a_1 = 22$.

(b) (i) Find the number of integer solutions of the equation

$$x_1 + x_2 + x_3 + x_4 + x_5 = 30$$

where $x_1 \geq 2,\ x_2 \geq 3,\ x_3 \geq 4,$
$x_4 \geq 2,\ x_5 \geq 0$

(ii) Given the in order and post order traversal of a tree T

In order : BEHFACDGI

Post order : HFEABIGDC

Determine the tree T and its pre order.

(c) (i) Prove that for any connected planar graph,

$$V - e + r = 2$$

where v, e, r are the number of vertices, edges and regions of the graph respectively.

(ii) Define and explain the following :

(a) Bipartite graph

(b) Chromatic number of a graph

(c) Binary search tree

(d) Adjacency matrix of a graph.
