Printed Pages : 4 **EOE038**

(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 0934 Roll No. | | | | | | | | | | |

B.Tech

(SEM III) ODD SEMESTER THEORY EXAMINATION 2009-10

DISCRETE MATHEMATICS

Time : 3 Hours] *[Total Marks : 100*

Note : *Attempt all the questions.*

1 Attempt any four parts of the following : **5×4=20**

(a) Give a combinatorial argument to show that for integers n, r with $n \geq r \geq 2$.

$$^{n+2}C_r = {}^nC_r + 2\,{}^nC_{r-1} + {}^nC_{r-2}.$$

(b) Define a relation $\mathbb{R}$ on a set $X = \{1, 2, 3, 4, 5\}$

(i) Which is only reflexive

(ii) Which is reflexive and symmetrics

(iii) Which is symmetric but not reflexive.

(c) Let $\mathbb{R} = \{(a, b), (a, d), (b, c), (b, d), (c, d)\}$ and $S = \{(a, a), (c, a), (d, c), (d, a), (d, b)\}$ be two relations on a set $X = \{a, b, c, d\}$.

Find $(\mathbb{R}oS)^5$ and $\mathbb{R}^5 oS^4$.

(d) Each user on a computer system has a password which is six to eight character long, where each character is an upper case letter or a digit. Each password must contain at least one digit. How many possible passwords are there ?

(e) How many positive integer between 1000 and 9999 inclusive are divisible by 7 and 11 ?

(f) Define a one-one onto function. Show that if f and g are one-one onto then fog is also one-one onto.

2 Attempt any two parts of the following : **10×2=20**

(a) Construct a truth table for each of the compound propositions :

(i) $(p \to q) \vee (\neg p \to \neg q)$

(ii) $(p \Leftrightarrow q) \vee (\neg p \to q)$.

(b) Show that the following premises are inconsistent :

(i) If Jack misses many classes through illness, then he fails high school.

(ii) If Jack fails high school, then he is uneducated.

(iii) If Jack reads a lot of books, then he is not uneducated.

(iv) Jack misses many classes through illness and reads a lot of books.

(c) Test the validity of the following argument :

If I study, then I will not fail mathematics;
If I do not play basket ball, then I will study.
<u>But I failed mathematics.</u>
Therefore, I must have played basket ball.

3 Attempt any four parts of the following : **5×4=20**

(a) If n is a non-negative integer, then

$$\sum_{k=0}^{n} \left({}^{n}C_k\right)^2 = {}^{2n}C_n.$$

(b) Suppose that k and n are integers with $1 \le k < n$, prove that

$${}^{n-1}C_{k-1} \cdot {}^{n}C_{k+1} \; {}^{n+1}C_k = {}^{n-1}C_k \cdot {}^{n}C_{k-1} \; {}^{n+1}C_{k+1}.$$

(c) Find the solution to the recurrence relation $a_n = 6a_{n-1} - 11a_{n-2} + 6a_{n-3}$ with the initial conditions $a_0 = 2, a_1 = 5, a_2 = 15$.

(d) Prove that for $(n+1)$ st fibonacci number

$$f_{n+1} = {}^{n}C_0 + {}^{n-1}C_1 + \cdots\cdots + {}^{(n-k)}C_k$$

where n is positive integer and $k = \left[\frac{n}{2}\right]$.

(e) Show that for positive integer n

$$\binom{-\frac{1}{2}}{n} = \binom{2n}{n} \Big/ (-4)^n.$$

(f) Show that the coefficient $p(n)$ of x^n in the formal power series expansion of $1/(1-x)(1-x^3)(1-x^5)\ldots\ldots$ equals to the number of partitions of n.

4 Attempt any four parts of the following : **5×4=20**

(a) Define a group. Verify whether the set of all integers $\mathbb{Z}$ forms a group with respect to difference.

(b) Let $M_2(\mathbb{R})$ be the set of all 2×2 non-singular matrices of real numbers, verify whether $M_2(\mathbb{R})$ forms a group with respect to matrix multiplication.

(c) Define a cyclic group. Prove that cyclic group is ABELIAN.

(d) Define a permutation group. Let S_3 be a permutation group on three letter set $\{a, b, c\}$. Find three nontrivial sub-groups of S_3.

(e) Let $u(8)=\{1, 3, 5, 7\}$ be a group with respect to multiplication modulo 8. Prove that every element of $u(8)$ is its own inverse.

(f) Define a Ring. Verify whether $\mathbb{Z}_p=\{0, 1, 2, 3, \ldots (p-1)\}$ p prime w.r.t. addition and multiplication modulo p.

5 Attempt any four parts of the following : 5×4=20

(a) Prove that the sum of the degrees of all the vertice of a graph is even.

(b) Find the example an eulerian graph which is also hamiltonian.

(c) Define the pendent vertices of a tree. Prove that every tree $T=T(V, E)$ with V vertices and E edges, $|V|\geq 2$ has at least two pendent vertices.

(d) Define a finite automation machine. Construct deterministic finite state automata that recognize each of these languages :

(i) the set of bit strings that begin with two oo's

(ii) the set of bit strings that end with two o's

(iii) the set of bit strings that contain at least two o's.

(e) Draw the transition diagram of non-deterministic finite state automata $M=(S, A, I, f, s_o)$ where

S \ I	a	b
s_0	$\{s_1\}$	$\{s_0\}$
s_1	$\{s_1\}$	$\{s_1, s_2\}$
s_2	ϕ	ϕ

, $S=\{s_0, s_1, s_2\}$

$A=\{s_1\}$