Printed Pages : 3 **TCS301**

(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 1064 Roll No. | | | | | | | | | | |

B.Tech

(SEM III) ODD SEMESTER THEORY EXAMINATION 2009-10

DISCRETE STRUCTURE

Time : 3 Hours] *[Total Marks : 100*

Note : *Attempt all questions.*

1 Attempt any **four** parts of the following : 5×4=20

(a) Show that $(R \subseteq S) \wedge (S \subset Q) \Rightarrow R \subset Q$. Is it correct to replace $R \subset Q$ by $R \subseteq QP$.. Explain your answer.

(b) Let $N = \{0, 1, 2, 3, ...\}$. Define functions f, g and h form set N to N by $f(n) = n+1$,

$$g(n) = 2n,\ h(n) = \begin{cases} 0 & \text{if n is even} \\ 1 & \text{if n is odd} \end{cases}$$

Compute *go (fog) oh*.
Is the function h is inversible ?
Is the function f is on to ?

(c) Given a covering of the set $S = \{A_1, A_2 ...., A_n\}$, show how you can write a compatibility relation which defines this covering.

(d) Let $f: X \to Y$ and $g: Y \to X$. Prove that the function g is equal to $f-1$ only if $gof = Ix$ and $fog = Iy$.

(e) Show that the predicate "x is prime" is primitive recursive.

(f) Show that $n3 + 2n$ is divisible by 3.

2 Attempt any **four** parts of the following : 5×4

(a) If G is a group in which $(ab)^i = a^i b^i$ for three consecutive integers i and any a, b in G, show that G is abelian.

(b) Show that the intersection of any two congruence relations on a set is also a congruence relation.

(c) Show that the relation of isomorphism is an equivalence relation.

(d) Show that every finite semigroup has an idempotent.

(e) Show that for any commutative monoid $\langle M, * \rangle$, the set of idempotent elements of M forms a submonoid.

(f) Write about cosets and permutation groups.

3 Attempt any **two** parts of the following : 10×2=20

(a) Give an example of a set X such that $\langle \rho(X), \subseteq \rangle$ is a totally ordered set.

(b) Prove that a n variable boolean function having products of all maxterm is zero.

(c) (i) Define Binary search tree. Show the insertion of an element in an existing binary search tree.

(ii) Prove that a tree with n vertices will have n-1 edges.

4 Attempt any **two** of the following parts : 10×2

(a) (i) Write the following statement in symbolic form. "If either Ram takes Maths or Shyam takes Science, then Hari will take Biology".

(ii) Construct the truth table for

$$(P \to Q) \wedge (Q \to P).$$

(b) Obtain formulas having the simplest possible form which are equivalent to formulas :

(i) $P \vee (\neg P \vee (Q \wedge \neg Q))$.

(ii) $(P \wedge (Q \wedge S)) \vee (\neg P \wedge (Q \wedge S))$.

(c) Show that $\neg P(a, b))$ follows logically from (x) (y) $(P(x, y) \to W(x, y))$ and $\neg W(a, b)$.

5 Attempt any **two** of the following parts : 10×2

(a) (i) Solve the recurrence relation $dn = 2\,dn - 1 - dn - 2$.

(ii) Write about linked list representation of graphs.

(b) Show that if G be a graph of n vertices and m edges then G has Hamiltonian circuit if

$$m \geq \frac{1}{2}(n^2 - 3n + 6).$$

(c) (i) Prove that a tree of connected graph has no circuit.

(ii) Define Euler graph. Give a suitable example for it.