(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 0111 **Roll No.** | | | | | | | | | |

B.Tech.

(SEM. III) ODD SEMESTER THEORY EXAMINATION 2010-11

DISCRETE MATHEMATICAL STRUCTURES

Time : 3 Hours *Total Marks : 100*

Note : (1) Attempt **all** questions.

(2) All questions carry equal marks.

1. Attempt any **four** parts of the following :— **(5×4=20)**

(a) Consider a universal set $U = \{x \mid x \text{ is an integer}\}$. Assume that $X = \{x \mid x \text{ is a positive integer}\}$, $Z = \{x \mid x \text{ is an even integer}\}$ and $Y = \{x \mid x \text{ is a negative odd integer}\}$. Find the following :

(i) $X - Y$

(ii) $X^C - Y$, where X^C is the complement of set X.

(b) Consider a set $S_K = \{1, 2, ..., K\}$. Find

$$\bigcup_{k=1}^{n} S_k \text{ and } \bigcup_{k=1}^{\infty} S_k.$$

(c) Let R be a relation on N, the set of natural numbers such that

$$R = \{(x, y) \mid 2x + 3y \text{ and } x, y \in N\}.$$

Find :

(i) The domain and codomain of R.

(ii) R^{-1}.

(d) Show that the functions $f(x) = x^3 + 1$ and $g(x) = (x - 1)^{1/3}$ are converse to each other.

(e) Prove that if f_n is a Fibonacci number then

$$f_n = \frac{1}{\sqrt{5}}\left[\left(\frac{1+\sqrt{5}}{2}\right)^{n+1} - \left(\frac{1-\sqrt{5}}{2}\right)^{n+1}\right]$$

for all $n \in N$, the set of natural numbers.

(f) Let $f : X \rightarrow Y$ and $X = Y = R$, the set of real number. Find f^{-1} if

(i) $f(x) = x^2$

(ii) $f(x) = (2x - 1)/5$.

2. Attempt any two parts of the following :— (10×2=20)

(a) Let G = {1, –1, i, –i} with the binary operation multiplication be an algebraic structure, where $i = \sqrt{-1}$.

(i) Determine whether G is an Abelian.

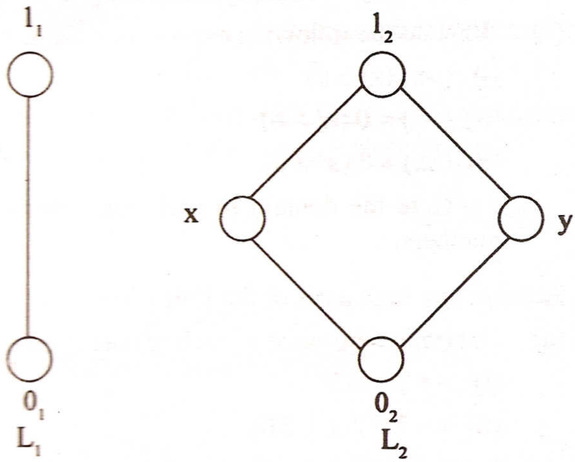
(ii) If G is a cyclic group, then determine the generator of G.

(b) Let $G = (Z^2, +)$ be a group and let H be a subgroup of G, where $H = \{(x, y) \mid x = y\}$. Find the left cosets of H in G. Here Z is the set of integers.

(c) Prove that (R, +, *) is a ring with zero divisors, where R is 2×2 matrix and + and * are usual addition and multiplication operations.

3. Attempt any **two** parts of the following :— **(10×2=20)**

(a) Let $(L_1, \leq)$ and $(L_2, \leq)$ be lattices as shown below. Then draw the Hasse diagram for the lattice $(L, \leq)$, where $L = L_1 \times L_2$.



(b) (i) Simplify the following Boolean function using K-map :

$$f(x, y, z) = \Sigma(0, 2, 3, 7).$$

(ii) How are sequential circuits different from combinational circuits ?

(c) Describe the Boolean duality principle. Write the dual of each Boolean equations :

(i) $x + \bar{x}y = x + y$

(ii) $(x \cdot 1)(0 + \bar{x}) = 0.$

4. Attempt any **two** parts of the following :— **(10×2=20)**

(a) (i) Show that the statements :

$P \rightarrow Q$ and $\neg Q \rightarrow \neg P$ are equivalent.

(ii) State the contrapositive and converse statement of the following statement :

"If the triangle is equilateral, then it is equiangular."

(b) Show that premises :

$P \rightarrow Q, R \rightarrow S, \neg Q \rightarrow \neg S, \neg\neg P.$

and $(T \wedge U) \rightarrow R$ imply the conclusion $\neg(T \wedge U)$.

(c) What do the following expressions mean ?

(i) $(\forall x)(x^2 \geq x)$

(ii) $(\forall x) < 0 \ (x^2 > 0)$

(iii) $(\exists x) \neq 0 \ (x^2 \neq 0)$.

Here the domain in each case consists of the real numbers.

5. Attempt any **four** parts of the following :— **(5×4=20)**

(a) Determine the value of each of these prefix expressions :

(i) $- * 2 / 933$

(ii) $+ - * 335 / \uparrow 232$.

(b) For which values of n do these graphs have an Euler cycle :

(i) K_n, a complete graph of n-vertices.

(ii) C_x, a cycle of n-vertices.

(c) Solve the recurrence relation :

$T(n) = 64T(n/4) + n^6$ where

$n \geq 4$ and a power of 4.

(d) Solve the recurrence relation :

$a_n = 3a_{n-1} + 4^{n-1}$

for $n \geq 0$ and $a_0 = 1$.

(e) Determine the number of bit strings of length 10 that either begin with three 0's or end with two 1's.

(f) How many different rooms are needed to assign 500 classes, if there are 45 different time periods during in the university time table that are available ?