(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 1064 Roll No. | | | | | | | | | | |

B.Tech.

(SEM. III) ODD SEMESTER THEORY EXAMINATION

2010-11

DISCRETE STRUCTURES

Time : 3 Hours *Total Marks : 100*

Note : (1) Attempt **all** questions.

(2) Make suitable assumptions wherever necessary.

1. Attempt any **four** parts of the following : **(5×4=20)**

(a) Prove that complement of the union of two sets is the intersection of their complements.

(b) The relation R on the set S of all real numbers is defined as : x R y if and only if $1 + xy > 0$. Show the different properties hold by the relation.

(c) What is meant by composition of two functions ? Let $f : R \rightarrow R$, and $g : R \rightarrow R$, where R is set of real numbers. Find f o g and g o f, where $f(x) = x^2 - 2$ and $g(x) = x + 4$.

(d) What is an equivalence relation ? Show that the relation of "similarity" on the set of all triangles in a plane is an equivalence relation.

(e) Prove the following property of Fibonacci numbers :

$f_1^2 + f_2^2 + \dots\dots\dots + f_n^2 = f_n f_{n+1}$; for all $n \geq 1$

where $f_0 = 0$, $f_1 = 1$ and $f_i = f_{i-1} + f_{i-2}$ for $i \geq 2$

(f) State and prove the pigeon hole principle.

2. Attempt any **four** parts of the following: **(5×4=20)**

(a) Let (S, *) be commutative semigroup. Show that if $a * a = a$ and $b * b = b$, then

$(a * b) * (a * b) = a * b$.

(b) Show that the set, $G = \{1, \omega, \omega^2\}$, where $1, \omega, \omega^2$ are the cube roots of the unity, form an abelian group under the operation of ordinary multiplication.

(c) Show that if G is a group such that $(ab)^n = a^n b^n$ for three consecutive integers, then $ab = ba$.

(d) Let G be the group of real numbers under addition, and let G^1 be the group of positive real numbers under multiplication. Let $f : G \to G^1$ be defined by $f(x) = e^x$. Determine whether f is an isomorphism or not.

(e) Define a subgroup. When a subgroup is said to be normal ? Explain with example.

(f) Define a field. How does a field differ from a ring ?

3. Attempt any **two** parts of the following : **(10×2=20)**

(a) (i) Define a Poset. Show that "inclusion" relation on set of subsets of a set is partial ordering.

(ii) Show that there are only five distinct Hasse diagrams for partially ordered sets that contain three elements.

(b) Define a distributive lattice. Show that the elements of the lattice $(N, \leq)$, where N is the set of positive integers and $a \leq b$ if and only if "a divides b" satisfy the distributive property.

(c) Obtain the product of sums and sum of product canonical forms of the Boolean expression $(X_1 \oplus X_2)' * X_3$.

4. Attempt any **two** parts of the following : **(10×2=20)**

(a) (i) Define the statement formula and well formed formula. Give some examples of each.

(ii) Construct the truth table for $(P \to Q) \wedge (Q \to P)$.

(b) (i) Show that $((P \vee \sim Q) \wedge (\sim P \vee \sim Q)) \vee Q$ is a tautology.

(ii) Show that $(\sim P \wedge Q) \Rightarrow (Q \Rightarrow P)$ is not a tautology.

(c) (i) Let P(x) : x is a person

F(x,y) : x is the father of y

M(x,y) : x is the mother of y

Write the predicate "x is the father of the mother of the y".

(ii) For the following set of premises, what relevant conclusion can be drawn ? Write down the rules of inference you used.

All foods that are healthy to eat do not taste good. Tofu is healthy to eat. You only eat what tastes good. You do not eat Tofu. Cheese burgers are not healthy to eat.

. Attempt any **two** parts of the following : **(10×2=20)**

(a) What is meant by connected graph ? Prove that a given connected graph G is an Euler graph if and only if all vertices of G are of even degrees.

(b) Define the following terms and explain with suitable example.

(i) Bipartite graph

(ii) Planar graph

(iii) Hamiltonian circuits

(iv) Homomorphism of graphs.

(c) Consider following recurrence relation :

$$T(n) = 7T(n/2) + 18n^2 \quad n \geq 2$$

$$T(1) = 1$$

Given than n is some power of 2.

Solve the given relation.