(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 9618   Roll No. ⬚⬚⬚⬚⬚⬚⬚⬚⬚⬚⬚

B.Tech.

(SEM. III) ODD SEMESTER THEORY EXAMINATION 2010-11

MATHEMATICS—III

Time : 3 Hours   *Total Marks : 100*

Note : (1)   Attempt **ALL** questions.

(2)   Provide Chi-square table.

1.   Attempt any **four** parts of the following :—   **(5×4=20)**

(a)   An electrostatic field in the xy-plane is given by the potential function $\varphi = 3x^2y - y^3$.

Find the stream function and hence find complex potential.

(b)   Show that the function $f(z)$ defined by

$$f(z) = \begin{cases} \dfrac{x^3y^5(x+iy)}{x^6+y^{10}}, & z \neq 0 \\ 0 & , & z = 0 \end{cases}$$

is not analytic at the origin even though it satisfies Cauchy-Riemann equations at the origin.

(c)   Expand $f(z) = \dfrac{z}{(z-1)(2-z)}$ in Laurent series valid for

(i) $|z-1| > 1$ and

(ii) $0 < |z-2| < 1$.

(d)   Verify Cauchy's theorem for the function $f(z) = 3z^2 + iz - 4$ along the perimeter of square with vertices $1 \pm i, -1 \pm i$.

(e)   Evaluate the following integral :

$$\oint_C \frac{12z-7}{(z-1)^2(2z+3)} dz,$$

where C is the circle $|z| = 2$.

(f) Using complex integration, evaluate $\int_0^\infty \frac{\cos mx}{1+x^2}\,dx$.

2. Attempt any **two** parts of the following :— **(10×2=20)**

(a) Calculate $\mu_1, \mu_2, \mu_3, \mu_4$ for the frequency distribution of heights of 100 students given in the following table and hence find coefficient of skewness and kurtosis.

Height (cm.)	144·5–	149·5–	154·5–	159·5–	164·5–	169·5–	174·5–
Class interval	149·5	154·5	159·5	164·5	169·5	174·5	179·5
Frequency	2	4	13	31	32	15	3

(b) Using method of least squares, derive the normal equations to fit the curve $y = ax^2 + bx$. Hence fit this curve to the following data.

x	1	2	3	4	5	6	7	8
y	1	1·2	1·8	2·5	3·6	4·7	6·6	9·1

(c) From the data given find the equation of lines of regression of x on y and y on x. Also calculate the correlation coefficient.

x	2	4	6	8	10
y	5	7	9	8	11

3. Attempt any **two** parts of the following :— **(10×2=20)**

(a) The demand for a particular spare part in a factory was found to vary from day-to-day. In a sample study, the following information was obtained :

Days	Mon	Tue	Wed	Thurs	Fri	Sat
No. of Parts Demanded	1124	1125	1110	1120	1125	1116

Use Chi-square to test the hypothesis that number of parts demanded does not depend on the day of the week at 5% level of significance.

(b) From the following series of annual data, find the trend line of semi-averages. Also estimate the value for 2009.

Year	2000	2001	2002	2003	2004	2005	2006	2007	2008
Actual Value	170	231	261	267	278	302	299	298	340

(c) Distinguish between the np-chart and p-chart. The following is data of defective of 10 samples of size 100 each. Construct $\bar{np}$-chart and give your comments.

Sample No.	1	2	3	4	5	6	7	8	9	10
No. of Defective	6	9	12	5	12	8	8	16	13	7

4. Attempt any **four** parts of the following :— **(5×4=20)**

(a) Perform five iterations of the bisection method to obtain the smallest positive root of the equation $x^3 - 5x + 1 = 0$.

(b) Find the real root of the equation $x \log_{10} x = 4.77$ correct to four decimal places using Newton-Raphson method.

(c) Find the number of men getting wage between Rs. 10 from the following table :

Wages (in Rs.)	5	15	25	35
No. of men	9	30	35	42

(d) Prove the following relations :

(i) $\mu\delta = \dfrac{\Delta E^{-1}}{2} + \dfrac{\Delta}{2}$

(ii) $\mu\delta = \dfrac{\nabla + \Delta}{2}$

(e) Use Newton's divided difference formula to find the interpolating polynomial and hence evaluate $y(9.5)$ from the given data :

x	7	8	9	10
y	3	1	1	9

(f)   Determine the missing values in the following table :

x	0	5	10	15	20	25
y	6	10	–	17	–	31

5.   Attempt any **two** parts of the following :—   (10×2=20)

(a)   Test if the following system of equations is diagonally dominant and hence solve this system using Gauss-Seidal method :

$$2x_1 + x_2 + 4x_3 = 7$$
$$3x_1 + x_2 + 2x_3 = 6$$
$$-x_1 + 4x_2 + 2x_3 = 5.$$

(b)   (i)   Compute $f'(3)$ from the following table :

x	1	2	4	8	10
f(x)	0	1	5	21	27

(ii)   The velocities of a car which starts initially from rest (running on a straight road) at intervals of 2 minutes are given below :

Time (minutes)	2	4	6	8	10	12
Velocity (km/hr)	22	30	27	18	7	0

Apply Simpson's 3/8 rule to find the distance covered by the car.

(c)   Estimate $y(1)$ if $2yy' = x^2$ and $y(0) = 2$ using Runge-Kutta method of fourth order by taking $h = 0.5$. Also compare the result with exact value.