Printed Pages : 7 **ECS-303**

(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 0111 Roll No. | | | | | | | | | | |

B. Tech.

(Semester-III) Theory Examination, 2011-12

DISCRETE MATHEMATICAL STRUCTURES

Time : 3 Hours] *[Total Marks : 100*

Note : Attempt questions from each Section as per directions.

Section-A

1. Attempt *all* parts of this question. 2×10=20

(a) Show that :

$$(A-B) \cap (B-A) = \phi.$$

(b) Show that :

$$\lfloor n/3 \rfloor + \lceil 2n/3 \rceil = n \text{ for all positive integer } n \geq 1.$$

(c) Show that the set $\{1, \omega, \omega^2\}$ is a cyclic group of order 3 with respect to multiplication, ω being the cube root of unity.

(d) Define Ring by giving a suitable example.

(e) Draw the Hasse diagram of the following set under the partial ordering relation "divides" :

$$\{2, 4, 8, 16\}.$$

(f) Simplify the following Boolean expression :

$$X = AB' + A'B' + A'B + AB.$$

(g) What do you mean by a tautology ? Give a suitable example.

(h) $P(x) : x$ is a person.

$F(x, y) : x$ is the father of y.

$M(x, y) : x$ is the mother of y.

Write the predicate "x is the father of the mother of y".

J

(i) Define a spanning tree. Give an example.

(j) Define Bipartite graph by giving a suitable example.

Section-B

2. Attempt any *three* parts of this question. 10×3=30

(a) (i) Prove by contradiction that $6-7\sqrt{2}$ is irrational.

(ii) $S=\{(1, 2), (2, 1)\}$ is a binary relation on set $A=\{1, 2, 3\}$. Is it irreflexive ? Add the minimum number of ordered pairs to S to make it an equivalence relation. Give the modified S.

(b) (i) Show that in a group $<G,*>$, if for any $a, b \in G$, $(a*b)^2=a^2*b^2$, then $<G,*>$ must be abelian.

(ii) Prove that the set E of all even integers is a commutative ring with respect to usual addition and multiplication, but it has no unit element.

(c) (i) Show that the operations of meet and join on a lattice are commutative, associative and idempotent.

(ii) Draw Karnaugh map and simplify the following boolean expression :

$$\overline{A}\,\overline{B}\,\overline{C}\,\overline{D}+\overline{A}B\overline{C}D+\overline{A}\,\overline{B}CD+\overline{A}\,\overline{B}C\overline{D}+\overline{A}BCD.$$

(d) (i) Determine whether each of the following is a tautology, a contradiction or neither :

(1) $A \leftrightarrow (A \vee A)$

(2) $(A \vee B) \rightarrow B$

(3) $A \wedge (\neg (A \vee B))$

(ii) Prove that :

$$(\exists x)\ (P(x) \wedge Q(x)) \Rightarrow (\exists x)\, P(x) \wedge (\exists x)\, Q(x).$$

(e) (i) Five balls are to be placed in three boxes. Each can hold all the five balls. In how many different ways can we place the balls so that no box remains empty, if :

(1) balls and boxes are all different

(2) balls are identical but boxes are different ?

(ii) Solve the recurrence relation :

$$a_n = a_{n-1} + 2, \quad n \geq 2$$

subject to the initial condition $a_1 = 3$.

Section-C

Attempt *all* questions of this Section. 10×5=50

3. Let A be a set of n elements. Let N_r be the number of binary relations on A and let N_f be the number of functions from A to A.

(a) Give the expression for N_r in terms of n.

(b) Give the expression for N_f in terms of n.

(c) Which is larger for all possible n, N_r or N_f.

4. (a) Prove that :

power set $(A \cap B)$ = power set $(A) \cap$ power set (B).

(b) Let sum$(n)=0+1+2+\ldots\ldots\ldots n$ for all natural numbers n. Give an induction proof to show that the following equation is true for all natural numbers m and n :

$$\text{sum}(m+n) = \text{sum}(m) + \text{sum}(n) + mn.$$

5. Prove that the order of a subgroup of a finite group divides the order of the group.

6. Prove that there are n^{n-2} labelled trees with n vertices.

7. Attempt any two parts :

(a) Show that there are only five distinct Hasse diagrams for partially ordered sets that contain three elements.

(b) Prove that every connected graph has at least one spanning tree.

(c) Show that a lattice with three or fewer elements is a chain.