(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 0934

Roll No. | | | | | | | | | |

B. Tech

(SEMESTER-III) THEORY EXAMINATION, 2012-13

DISCRETE MATHEMATICS

Time : 3 Hours] *[Total Marks : 100*

Section – A

1. Attempt **all** parts. **$10 \times 2 = 20$**

(a) Let $S = \{2, 5, \sqrt{2}, 25, \Pi, 5/2\}$ and $T = \{4, 25, \sqrt{2}, 6, 3/2\}$

Find $S \cap T$ and $T \times (S \cap T)$

(b) Let $A = \{0, 1, 2, 3, \ldots\}$. Define function f, g and h from A to A by $f(x) = 2x$, $g(x) = x + 1$ and

$$h(x) = \begin{cases} 0, \text{ if } x \text{ is odd} \\ 1, \text{ if } x \text{ is even} \end{cases}$$

where $x \in A$. (Treat '0' as an even number). Find $(f \circ g) \circ h$.

(c) Let p be "He is tall" and let q "He is handsome". Then represent the statement. "It is false that he is short or handsome" in predicate logic.

(d) Write contra-positive of statement P : If the flood destroys my house or the fire destroys my house, then my insurance company will pay me.

(e) How many numbers of diagonals can be drawn by joining the vertices of an octagon ?

(f) In how many ways can a party of 7 persons arrange themselves around a circular table ?

(g) Let (A, *) be an algebraic system, where * is a binary operation such that for any a and b in A, $a * b = a$. Show that this operation is associative.

(h) Let f be the permutation defined on the set {1, 2, 3, 4, 5, 6} by $f = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 1 & 5 & 6 & 3 & 4 \end{pmatrix}$. Express f as a product of disjoint cycles.

(i) Find the regular expression over {0, 1} that denotes the set of all strings not containing 100 as sub string.

(j) Find the number of possible binary trees with 4 nodes.

Section – B

2. Attempt any **three** parts. **$10 \times 3 = 30$**

(a) In a group of athletic teams in a certain institute, 21 are in the basket ball team, 26 in the Hockey team, 29 in the Football team. If 14 play Hockey and Basketball, 12 play Football and Basketball, 15 play Hockey and Football, 8 play all the three games.

(i) How many players are there in all ?

(ii) How many play only Football ?

Use Venn diagram to justify your answer.

(b) Test the validity of argument :

"If it rains tomorrow, I will carry my umbrella, if its cloth is mended. It will rain tomorrow and the cloth will not be mended. Therefore I will not carry my umbrella".

(c) Find the number of ways in which 5 prizes can be distributed among 5 students such that

(i) Each student may get a prize.

(ii) There is no restriction to the number of prizes a student gets.

(d) (i) Let G be any group in which every element is its own inverse. Show that G is abelian.

(ii) Let (a, *) be a commutative semi-group, show that if a * a = a and b * b = b, then (a * b) * (a * b) = a * b.

(e) Explain Euler graph using suitable example. Prove that a given connected graph G is an Euler graph if all vertices of G are of even degree.

Section – C

Attempt **all** parts. **$10 \times 5 = 50$**

3. Attempt any **two** parts :

(a) Prove that if a set A contains n elements. Then P(A) contains 2^n elements.

(b) Let A = {–2, –1, 0, 1, 2}, B = {0, 1, 4} and f : A $\rightarrow$ B is defined as $f(x) = x^2$ is a function. If so find that whether it is one to one or bijection ?

(c) If A and B are two subsets of a universal set then prove that

$$A - B = A \cap \overline{B}$$

4. Attempt any **two** parts :

(a) Is the proposition S a valid conclusion from the premises :

$P \Rightarrow q, p \Rightarrow r, \sim (q \vee r)$ and $s \vee p$?

(b) Prove that $(p \leftrightarrow q) \leftrightarrow r = p \leftrightarrow (q \leftrightarrow r)$

(c) Show that $((p \vee \sim q) \wedge (\sim p \vee \sim q)) \vee q$ is a tautology, where p and q are Boolean variables.

5. Attempt any **two** parts :

(a) In the word 'MANORAMA'.

(i) Find the number of permutations formed taking all letters.

(ii) Out of these the number of permutations with all A's together.

(b) Prove by induction that for all integers $n \geq 4$, $3^n > n^3$.

(c) If Fn satisfied the Fibonacci relation for the Fibonacci series (1, 1, 2, 3,.....) defined by the recurrence relation, Fn = Fn – 1 + Fn – 2, F0 = F1, then find a formula to find n^{th} Fibonacci number.

6. Attempt any **two** parts :

(a) Let ({a, b}, ∗) be a semigroup where $a * a = b$, Show that

(i) $a * b = b * a$

(ii) $b * b = b$

(b) Let G be the set of all non-zero real numbers and let $a * b = ab/2$, show that (G, ∗) is an abelian group.

(c) In S3, find two permutations f and g that do not commute.

7. Attempt any **two** parts :

(a) Design an automation which accepts only even numbers of 0s and even number of 1's.

(b) Suppose the characters ‘a’, ‘b’, ‘c’, ‘d’, ‘e’, ‘f’, ‘g’ are stored in a Binary Search Tree (BST). Draw a BST that is as tall as possible and contains all these characters. Also draw a BST that is as short as possible and contains all characters.

(c) Let M be a FSM given by following table. Construct an equivalent FSM corresponding to the machine M.

State	Input		Output
	1	2	
A	B	C	0
B	F	D	0
C	G	E	0
D	H	B	0
E	B	F	1
F	D	H	0
G	E	B	0
H	B	C	1