(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 9618 Roll No. | | | | | | | | | | |

B.Tech.

(SEMESTER-III) THEORY EXAMINATION, 2012-13

MATHEMATICS – III

Time : 3 Hours] *[Total Marks : 100*

Note : (1) All Questions are compulsory.
(2) Assume suitable data wherever necessary.
(3) Draw neat figure wherever necessary.

Section – A

1. Answer the following :

(a) Using the Cauchy Riemann equations show that $f(z) = |z|^2$ is not analytical at any point. 2

(b) Define singular point of an analytical function. Find nature and location of the singularity of $f(z) = \frac{z - \sin z}{z^2}$. 2

(c) Define skewness, coefficient of skewness, Kurtosis and coefficient of Kurtosis. 2

(d) Two events A and B have probabilities 0.25 and 0.50 respectively. The probability that both events A and B occur in 0.14. Find the probability that neither A nor B occurs. 2

(e) If the sum of the mean and variance of a Binomial distribution of 5 trials is $\frac{9}{5}$, find $P(X \geq 1)$. 2

(f) It has been found that 2% of the tools produced by a certain machine are defective. What is the probability that in a shipment of 400 such tools 3% or more will be defective ? 2

(g) Derive Newton Raphson formula to find approximate root of the equation $f(x) - 0$. 2

(h) Prove that $e^x = \left(\frac{\Delta^2}{E}\right) e^x \cdot \frac{Ee^x}{\Delta^2 e^x}$ 2

(i) Derive Newton-Cote's quadrature formula for numerical integration. 2

(j) Using Taylor's series method, find an approximate value of $y(0.1)$ if $y' = y^2 + x$, $y(0) = 1$. 2

Section – B

2. Answer any **three** of the following :

(a) (i) Evaluate the integral where $\int_C (z - z^2)\, d_z$ where C is the upper half of the circle $|z - 2| = 3$. What is the value of the integral if C is the lower half of the circle ? **6**

(ii) State and prove Cauchy's theorem. **4**

(b) (i) Define Moment generating function and two properties of moment generating function with proof. **4**

(ii) The probability density function of the random variable X is $f(x) = \frac{1}{2\theta} \exp\left(-\frac{|x-\theta|}{\theta}\right) - \infty < x < \infty$. Find moment generating function of X. **6**

(c) (i) Show that Poisson Distribution is a limiting case of Binomial Distribution. **4**

(ii) An insurance company insures 4000 people against loss of both eyes in a car accident. Based on previous data it was assumed 10 persons out 1,00,000 will have such type of injury in car accident. What is probability that more than 2 of the insured will collect on their policy in a given year ? **6**

(d) (i) Discuss rate of convergence of the Regula-falsi method. **4**

(ii) Find the cubic polynomial which takes the following values : **6**

X	3	4	5	6
Y	6	24	60	124

(e) (i) Solve the system of equations : **6**

$20x + y - 2z = 17$, $3x + 20y - z = -18$, $2x - 3y + 20z = 25$ by Gauss Seidel method.

(ii) A solid of revolution is formed by revolving the area $y = f(x)$ from $x = 1$ to $x = 3$ about x axis. The following table gives relation between x and y for this curve : **4**

X	1.0	1.5	2	2.5	3
Y	1.0000	0.9896	0.9589	0.9089	0.8415

Using Sympson's one-third rule estimate the volume the solid formed.

Section – C

3. Answer the following :

(a) If $f(z) = \dfrac{z+4}{(z+3)(z-1)^2}$, find Laurent's series expansion in $0 < |z-1| < 4$ and $|z-1| > 4$. **10**

OR

Show that $\int_C^{\infty} \dfrac{x^{p-1}}{1+x}\, dx = \dfrac{\pi}{\sin px}$, $0 < p < 1$

(b) Estimate Y at X = 5 by fitting a least square curve of the form $Y = \dfrac{b}{X(X-a)}$ to the following data : **10**

X	3.6	4.8	6.0	7.2	8.4	9.6	10.8
Y	0.83	0.31	0.17	0.10	0.07	0.05	0.04

OR

If θ is the angle between the two regression lines, then express $\tan\theta$ in terms of correlation coefficient (r). Explain the significance when $r = 0$ and $r = \pm 1$. Show that correlation coefficient is the geometric mean between two regression coefficient.

(c) A survey of 320 family with 5 children each revealed the distribution shown in the following table : **10**

No. of boys & girls	5 boys 0 girls	4 boys 1 girl	3 boys 2 girls	2 boys 3 girls	1 boy 4 girls	5 boys 0 girls	Total
No. of families	18	56	110	88	40	8	320

Is the result consistent with hypothesis that male and female births are equally probable ?

OR

In a certain factory there are two independent process of manufacturing the same item. The average weight in a sample of 250 items produced from one process is found to be 120 gram with standard deviation 12 gram, while the corresponding figures in a sample of 400 items from the other process are 124 gram and 14 gram respectively. Find standard error of difference between the two sample means. Is there significant difference in items produced by two processes ? Find 99% confidence limits for the difference in the average weights of the items produced by the two processes respectively.

(d) Find Newton's divided differences polynomial for the following data : 10

x	–3	–1	0	3	5
f(x)	–30	–22	–12	330	3458

Also find f(2) using Lagrange's interpolation formula.

OR

Derive Newton's interpolation formulae and using suitable Newton's formula, estimate the number of students who obtained marks between 40 and 45 given that :

Marks	30 – 40	40 – 50	50 – 60	60 – 70	70 – 80
No. of Students	31	42	51	35	31

(e) The speed of a train at various times after leaving one station until it stops at another station are given in the following table : 10

Speed in mph	0	13	33	39.5	40	40	36	15	0
Time in minutes	0	0.5	1	1.5	2	2.5	3	3.25	3.5

Find the distance between the two station using trapezoidal rule, Simpsons '(1/3) rule, Simpson's (3/8) rule, and Weddle's rule.

OR

Using fourth order Runge-Kutta method, solve the initial value problem $\frac{d^2y}{dx^2} - x\left(\frac{dy}{dx}\right) + y^2 = 0$ with initial conditions y = 1, and $\frac{dy}{dx} = 0$ when $x = 0$ in the interval [0, 0.2] and step size h = 0.1.