(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 9618 Roll No. | | | | | | | | | | |

B.Tech.

(SEM. III) ODD SEMESTER THEORY EXAMINATION 2012-13

MATHEMATICS—III

Time : 3 Hours *Total Marks : 100*

Note : *Attempt **all** questions. Provide table for area under normal curve and chi-square.*

1. Attempt any **four** parts of the following : **(5×4=20)**

(a) If $f(z)=\dfrac{x^3y(y-ix)}{x^6+y^2}$ when $z\neq 0$

$= 0$ when $z=0$

Prove that $\dfrac{f(z)-f(0)}{z}\to 0$ as $z\to 0$ along any radius vector but not as $z\to 0$ in any manner.

(b) If $u = 3x^2y - y^3$ find the analytic function $f(z) = u + iv$.

(c) Evaluate the following integral using Cauchy's integral formula $\oint_C \dfrac{dz}{z^2(z^2-4)e^z}$ where C is the circle $|z| = 1$.

(d) Verify Cauchy's theorem by integrating e^{iz} along the boundary of the triangle with the vertices at the points $1 + i, -1 + i$ and $-1 - i$.

(e) Expand $f(z)=\dfrac{z}{(z-1)(2-z)}$ in Laurent series valid for (i) $|z-1|>1$ and (ii) $0<|z-2|<1$.

(f) Use Contour integral to evaluate $\int_0^{2\pi} \frac{d\theta}{3-2\cos\theta+\sin\theta}$.

2. Attempt any **two** parts of the following : **(10×2=20)**

(a) Find all four central moments and discuss skewness and Kurtosis and also Karl Pearson skewness for the frequency distribution given below :

Range of Expend. in Rs. (100)/month	2–4	4–6	6–8	8–10	10–12
No. of Families	38	292	389	212	69

(b) Find the m.g.f. of the random variable X having the following probability density function

$$F(x)=\begin{cases} x & 0\le x\le 1 \\ 2-x & 1\le x<2 \\ 0 & \text{otherwise} \end{cases}$$

Also find mean and variance of X.

(c) Calculate the Rank Coefficient from the sales and expenses of 10 firms as given below :

Sales X	45	56	39	54	45	40	56	60	30	36
Expenses Y	40	36	30	44	36	32	45	42	20	36

3. Attempt any **two** parts of the following : **(10×2=20)**

(a) Suppose the weight W of 600 male students are normally distributed with mean $\mu = 70$ kg and standard deviation $\sigma = 5$ kg. Find (i) No. of students with weight between 69 and 74 kg. (ii) More than 76 kg.

(b) In a survey of 200 boys of which 75 were intelligent, 40 had educated fathers, while 85 of the unintelligent had uneducated father. Does this figure support the hypothesis that educated fathers have intelligent boys (Use chi-square test).

(c) Calculate the trend values by the method of least square from the data given below and estimate the sales for the year 2005 :

Year	1996	1997	1998	1999	2000
Sales of TV (in 000)	12	18	20	23	27

4. Attempt any **four** parts of the following : **(5×4=20)**

(a) Use Regula-Falsi method to find a positive root of the equation $\tan x + \tanh x = 0$ correct to six significant digits.

(b) Find the rate of convergence of Newton-Raphson method.

(c) Derive the following relationships :

(i) $\Delta = \frac{\delta^2}{2} + \delta\sqrt{1+\frac{\delta^2}{4}}$

(ii) $1+\delta^2\mu^2 = \left(1+\frac{1}{2}\delta^2\right)^2$

(d) Obtain the missing terms in the following table :

x	1	2	3	4	5	6	7	8
f(x)	2	4	8	–	32	–	128	256

Explain why the results differ from 16 and 64.

(e) Applying Lagrange's interpolation formula, find a cubic polynomial which approximates the following data :

x	−2	−1	2	3
y(x)	−12	−8	3	5

(f) Find the interpolating polynomial to the following data and hence find the value of y for x = 5 :

x	4	6	8	10
f(x)	1	3	8	16

5. Attempt any **two** parts of the following : **(10×2=20)**

(a) Solve the following system using Crout's decomposition method :

$$3x - y + 2z = 12$$
$$x + 2y + 3z = 11$$
$$2x - 2y - z = 2$$

(b) (i) From the following table, estimate $y'(1.05)$:

x	1.00	1.05	1.10	1.15	1.20	1.25
y	1.10000	1.1347	1.1688	1.1564	1.2344	1.2345

(ii) Evaluate the integral

$$\int_0^1 e^{x+1}\,dx$$

using Simpson's 1/3 rule, by dividing the interval of integration into eight equal parts.

(c) Use the Runge-Kutta Method of order four to solve

$$10\frac{dy}{dx} = x^2 + y^2,\ y(0) = 1$$

for the interval $0 < x \leq 0.2$ with $h = 0.1$ (two steps).