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(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 1225 Roll No. | | | | | | | | | | |

B.Tech.

(SEM. III) ODD SEMESTER THEORY

EXAMINATION 2013-14

MATHEMATICS – III

[Branches : EC, ET, EL, CS, IT, AEI, EI, IC EN, EE]

Time : 3 Hours *Total Marks : 100*

Note :– Attempt **all** questions from each Section as indicated. The symbols have their usual meaning.

SECTION–A

1. Attempt **all** parts of this Section. Each part carries 2 marks : **(2×10=20)**

(a) Find residue of $f(z)=\frac{2z+1}{z^2-z-2}$ at the pole z = –1.

(b) Define harmonic function.

(c) State Convolution theorem for Fourier Transform.

(d) Find the Z-Transform of $\{n_{C_k}\}$, $0 \le k \le n$.

(e) Define coefficients of kurtosis.

(f) Define marginal and conditional distribution.

(g) Prove that : $|(X, Y)| < \|X\| \, \|Y\|$.

(h) Define Abelian group.

(i) Define rate of convergence.

(j) Write the formula for Simpson's 3/8 rule.

SECTION–B

Note :– Attempt any **three** parts of this Section. **(10×3=30)**

2. (a) Apply calculus residues to prove that :

$$\int_0^\infty \frac{\cosh ax}{\cosh \pi x} dx = \frac{1}{2}\sec\frac{a}{2}.$$

(b) If $F_c(p) = \frac{1}{2}\tan^{-1}\frac{2}{p^2}$, then find $f(x)$.

(c) Show that Poisson distribution is a limiting form of binomial distribution when p is a very small and n is very large. Also find mean and variance of Poisson distribution.

(d) If $p = p(x) = p_0 + p_1x + p_2x^2$ and $q = q(x) = q_0 + q_1x + q_2x^2$, then the inner product is defined by :

$(p, q) = p_0q_0 + p_1q_1 + p_2q_2$ for the vectors $X_1 = 1 + 2x + 3x^2$ $X_2 = 3 + 5x + 5x^2$, $X_3 = 2 + x + 8x^2$. Find the orthogonal vectors.

(e) Use Gauss-Seidel method to solve the following system of simultaneous equations :

$$83x + 11y - 4z = 95$$

$$7x + 52y + 13z = 104$$

$$3x + 8y + 29z = 71$$

Perform four iterations.

SECTION–C

Note :– All questions of this Section are compulsory. Attempt any **two** parts from each question : **(10×5=50)**

3. (a) In a two dimensional fluid flow, the stream function is $\psi = -\frac{y}{x^2 + y^2}$, find the velocity potential Φ.

(b) Expand $f(z)=\frac{z}{(z-1)(z-2)}$ in Laurent series valid for region :

(i) $|z-1| > 1$

(ii) $0 < |z-2| < 1$.

(c) State and prove Cauchy's Theorem.

4. (a) Find Fourier cosine transform of $\frac{1}{1+x^2}$ and hence find Fourier sine Transform of $\frac{x}{1+x^2}$.

(b) Find the inverse Z-transform of :

$$F(z)=\frac{9z^3}{(z-2)(3z-1)^3}.$$

(c) Solve by Z-transform the difference equation :

$$y_{k+2} - 2y_{k+1} + y_k = 3k + 5. \quad y(0) = 0, y(1) = 1.$$

5. (a) In a certain factory manufacturing razor blades, there is a small chance of 0.002 for any blade to be defective. The blades are supplied in packets of 10. Use suitable distribution to calculate the approximate number of packets containing no defective, one defective and two defective blades respectively in a consignment of 20,000 packets.

(b) Find the moment generating function of the exponential distribution :

$$f(x)=\frac{1}{c}e^{-x/c}, 0 \le x \le \infty, c > 0.$$ Hence find its mean and S.D.

(c) Calculate the first four moments about the mean for the following data :

Class Interval	0-10	10-20	20-30	30-40	40-50
Frequency	10	20	40	20	10

6. (a) Examine the following vectors for linear dependence and find the relation, if it exists :

$X_1 = (1, 2, 1), X_2 = (3, 1, 5), X_3 = (3, -4, 5)$.

(b) Let V be the vector space of all real valued continuous functions over R. Then show that the solutions set W of the differential equation :

$3\frac{d^2y}{dx^2} + 11\frac{dy}{dx} - 4y = 0$, is a subspace of V.

(c) Show that the intersection of any two subspaces of a vector space is also a space of the same.

7. (a) Use Newton's Divided difference formula to find f(x) from the following data :

x	0	1	2	4	5	6
f(x)	1	14	15	5	6	19

(b) Compute the rate of convergence of Newton-Raphson method.

(c) Apply Runge-Kutta fourth order method to find an approximate value of y when x = 0.2, given that $\frac{dy}{dx} = x + y$ with initial condition y = 1 at x = 0.