(Following Paper ID and Roll No. to be filled in your Answer Book)

**PAPER ID : 1250** **Roll No.** | | | | | | | | | | |

**B.Tech.**

(SEM. III) ODD SEMESTER THEORY

EXAMINATION 2013-14

## SIGNALS AND SYSTEMS

*Time : 3 Hours* *Total Marks : 100*

**Note :**–Answer **all** the questions.

## SECTION–A

1. Attempt **all** parts : **(10×2=20)**

(a) Determine the fundamental period of the signal :

$x(t) = 3\sin(7t + 2) - 4\cos(4t + 1)$.

(b) Consider a discrete-time system with input x [n] and output y [n] :

$y[n] = x[n + 2] - x[n - 2]$.

Is this system Linear ?

(c) Determine the Z-Transform of $x[n] = a^{-n}u[-n]$.

(d) Find Laplace Transform of $x(t) = \sum_{k=0}^{\infty} \delta(t - kT)$

(e) Prove the time scaling property of Fourier transform.

(f) For the following frequency response of a causal and stable LTI System :

$$H(jw) = \frac{1 - jw}{1 + jw}.$$

Show that $|H(jw)| = A$, and determine the value of A.

(g) Consider a LTI System with step response $y(t) = e^{-t} u(t)$. Determine the output of this system to the input $x(t) = u(t-1) - u(t-3)$.

(h) Find the Fourier transform of the Signal :

$X(t) = e^{\alpha t} u(-t), a > 0.$

(i) $X(s) = \frac{s^2 + 5s + 7}{s^2 + 3s + 2}$, Determine the value of $x(\infty)$.

(j) Sketch the given signal :

$x(t) = r(t) u(3 - t).$

## SECTION–B

2. Attempt any **three** parts : **(3×10=30)**

(a) (i) Determine the impulse response of the Discrete Time System :

$y(n) - 3y(n-1) + 2y(n-2) = x(n) + 3x(n-1) + 2x(n-2).$

(ii) Let $x(t) = u(t-3) - u(t-5)$ and $h(t) = \bar{e}^{3t} u(t)$. Compute $y(t) = x(t) * h(t)$.

(b) (i) Define Ideal frequency Selective filter. Explain time domain properties of ideal frequency selective filter.

(ii) State and prove Sampling theorem and discuss the effect of under sampling.

(c) (i) Find the Inverse Laplace Transform of :

$$X(s) = \frac{2}{(s+4)(s-1)}$$ If the region of convergence is :

(a) $-4 < \text{Re}(s) < 1$

(b) $\text{Re}(s) > 1$

(c) $\text{Re}(s) < -4$.

(ii) Sketch and determine the convolution of the following signals :

$$x(t) = \pi\left(\frac{t-1}{3}\right) \; ; \; h(t) = u(t-5).$$

(d) (i) Find the Unilateral Z-Transform of :

$x[n] = [a^n \text{ Sin } w_o n]\, u[n]$.

(ii) If $x(z) = \frac{2z}{3z^2 - 4z + 1}$, find $x(n)$ ; $n \geq 0$. Given that ROC of $x(z)$ is $|z| > 1$.

(e) Realize $H(s) = \frac{s(s+2)}{(s+1)(s+3)(s+4)}$, by canonic direct form I and direct form II.

## SECTION–C

**Note :**—Attempt **all** questions in this Section.

3. Attempt any **one** part : **(5×10=50)**

(a) Impulse-Train sampling of x[n] is used to obtain :

$$g[n] = \sum_{k=-\infty}^{\infty} x[n] \ \delta[n-KN].$$

If $x(e^{jw}) = 0$ for $\frac{3\pi}{7} \leq |w| \leq \pi$, determine the largest value for the sampling interval N which ensures that no aliasing takes place while sampling x [n].

(b) Determine whether or not each of the following continuous-time signals are periodic. If the signal is periodic, determine its fundamental period :

(i) $x(t) = 3\cos(4t + \pi/3)$

(ii) $x(t) = e^{j(\pi t - 1)}$

(iii) $x(t) = [\cos(2t - \pi/3)]^2$

(iv) $x(t) = \cos^2 \frac{\pi}{8} t$

(v) $x(t) = \sum_{n=-\infty}^{\infty} e^{-(2t-n)} u(2t-n)$.

4. Attempt any **one** part :

(a) Find the Z-Transform x(z) and sketch the pole-zero plot with the ROC of following sequence :

$$x[n] = \left(\frac{1}{3}\right)^n u[n] + \left(\frac{1}{2}\right)^n u[-n-1].$$

(b) Using the power series expansion technique, find the inverse Z-Transform of the following x (z) :

(i) Using Long Division Method :

$$X(z)=\frac{z^2+2z}{z^3-3z^2+4z+1} \quad ROC|z|>1$$

(ii) $X(z) = \log_e(1 + az^{-1})$ ROC $|z| > a$.

5. Attempt any **one** part :

(a) Consider a continuous-time LTI System described by :

$$\frac{dy(t)}{dt} + 2y(t) = x(t)$$

Using the Fourier transform, find the output y (t) of the following input signal :

$x(t) = e^{-t}u(t)$ and determine the frequency response $H(e^{jw})$ of the system.

(b) Consider the signal x(t) in figure 1.

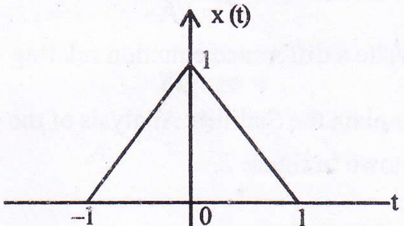


**Figure 1**

(i) Find the Fourier transform x (jw) of x (t).

(ii) Sketch the signal :

$$y(t) = x(t) * \sum_{k=-\infty}^{\infty} \delta(t-4k).$$

6. Attempt any **one** part :

(a) Consider two Right-Sided Signals x(t) and y(t) related through the differential equations :

$$\frac{dx(t)}{dt} = -2y(t) + \delta(t) \text{ and } \frac{dy(t)}{dt} = 2x(t).$$

Determine y(s) and x(s) alongwith their regions of Convergence.

(b) The input x [n] and output y [n] of a causal LTI system are related through the block diagram representation shown in figure 2 :

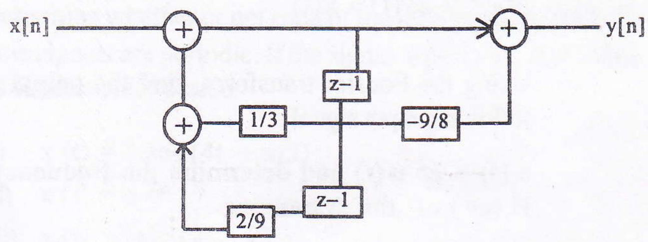


**Figure 2**

(i) Write a difference equation relating y [n] and x [n].

(ii) Explain the Stability Analysis of the given system as shown in figure 2.

7. Attempt any **two** parts :

(a) Let x (t) be a signal with Nyquist rate $w_o$. Determine the Nyquist rate for the following signals :

(i) x (t) + x (t − 1)

(ii) $x^2$ (t).

(b) Suppose that $x(t) = e^{-(t-2)} u(t-2)$ and h (t) is shown in figure 3 :

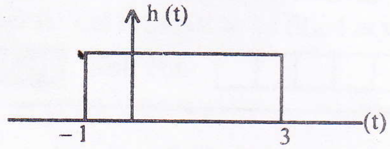


**Figure 3**

Verify the convolution property for this pair.

(c) Obtain the Fourier series for the wave form shown in figure 4 :

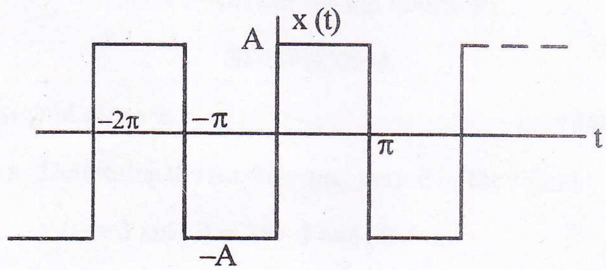


**Figure 4**