(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 1247 Roll No.

B.Tech.

(SEM. III) ODD SEMESTER THEORY EXAMINATION 2013-14

DISCRETE STRUCTURES

Time : 3 Hours *Total Marks : 100*

Note :–Attempt **all** Sections.

SECTION–A

1. Attempt all parts : **(10×2=20)**

(a) Let A = {a, {a}}. Determine whether the following statements are true or false :

(i) $\{a, \{a\}\} \in P(A)$

(ii) $\{a, \{a\}\} \subseteq P(A)$

(iii) $\{\{\{a\}\}\} \in P(A)$

(iv) $\{\{\{a\}\}\} \subseteq P(A)$.

(b) Find out the cardinality of the following sets :

A = {x : x is weeks in a leap year}

B = {x : x is a +ve divisor of 24 and not equal to zero}

C = $\{\{\{\}\}\}$

D = $\{\{\varnothing, \{\varnothing\}\}\}$.

(c) How many symmetric and reflexive binary relations are possible on a set S with cardinality n ?

(d) Define transitive closure with suitable example.

(e) Find the minimum number of students in a class to show that five of them are born on same month.

(f) Find the total number of squares in a chessboard.

(g) Define Group with suitable example.

(h) Define Lagrange's theorem. What is the use of the theorem ?

(i) Determine by means of truth table the validity of DeMorgan's theorem for three variables :

$(ABC)' = A' + B' + C'$.

(j) Define Binary Tree Traversal with example.

SECTION–B

2. Attempt all parts : **(3×10=30)**

(a) Let (A, ≤) be a partially ordered set. Let ≤ be a binary relation on A such that for a and b in A, a is related to b iff b ≤ a.

(i) Show that ≤ partially ordered relation.

(ii) Show that (A, ≤) is lattice or not.

(b) (i) Define cyclic group with suitable example.

(ii) Simplify the following Boolean functions using three variable maps :

(a) $F(x, y, z) = \Sigma (0, 1, 5, 7)$

(b) $F(x, y, z) = \Sigma (1, 2, 3, 6, 7)$

(c) (i) Show that in a connected planner linear graph with 6 vertices and 12 edges, each of the regions is bounded by 3 edges.

(ii) Show that a regular binary tree has an odd number of vertices.

SECTION–C

3. Attempt all parts : (5×10=50)

(a) Let A = {2, 3, 6, 12, 24, 36} and relation ≤ be such that 'x ≤ y' iff x divides y. Draw Hasse Diagram and find minimal and maximal elements.

(b) Find the number of integers between 1 and 250 that are divisible by any of the integers 2, 3, 5, and 7.

(c) Solve the following recurrence relation :

(i) $a_r - 7a_{r-1} + 10a_{r-2} = 0$, given that $a_0 = 0$ and $a_1 = 3$.

(ii) Given that $a_0 = 0$, $a_1 = 1$, $a_2 = 4$ and $a_3 = 12$ satisfy the recurrence relation $a_r + C_1a_{r-1} + C_2a_{r-2} = 0$, determine a_r.

OR

Prove by using mathematical induction that :

$7 + 77 + 777 + \ldots\ldots\ldots\ldots + 777\ldots7 = 7/81[10^{n+1} - 9n - 10]$, for every $n \in N$.

(d) (i) Given that the value of $P \rightarrow \overline{Q}$ is true, can you determine the value of $P \vee (P \leftrightarrow Q)$.

(ii) Construct the truth table for the following statements :

$(P \rightarrow \overline{Q}) \rightarrow \overline{P}$

$P \leftrightarrow (\overline{P} \vee \overline{Q})$.

OR

Let $A = \{1, 2, 3, 4\}$ and $R = \{(1, 2), (4, 3), (2, 2), (2, 1), (3, 1)\}$ is a relation defined on A. Find Transitive closure of R using Warshall's algorithms.

(e) (i) Suppose G is a finite cycle-tree graph with at least one edge. Show that G has at least two vertices of degree 1.

(ii) Show that a connected graph with n vertices must have at least $(n-1)$ edges.