



(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 199320

Roll No.

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B. Tech.

(SEM. III) (ODD SEM.) THEORY
EXAMINATION, 2014-15

MATHEMATICS-III

Time : 3 Hours]

[Total Marks : 100

Note : Attempt All Questions. All Questions carry equal marks

1 Attempt any four parts of the following : $5 \times 4 = 20$

(a) State Cauchy-Riemann theorem for an analytic function.

Test the analyticity of the following Function :

$$f(z) = \frac{(x^3 - y^3) + i(x^3 + y^3)}{x^2 + y^2}, \text{ if } z \neq 0 \text{ and}$$

$$= 0 \text{ if } z = 0$$

(b) State Cauchy- integral theorem for an analytic function. Verify this theorem by integrating the function $z^3 + iz$ along the boundary of the rectangle with vertices $+1, -1, i, -i$.

(c) Show that the function $u = \frac{1}{2} \log(x^2 + y^2)$ is harmonic.

Find the harmonic conjugate of u.

(d) Evaluate the integral $\int \frac{e^{2z}}{(z+1)^5} dz$, around the boundary

of the circle $|z| = 2$.

(e) Find the Taylor series expansion of the function $\tan^{-1} z$ about the point $z = \pi/4$.

(f) Evaluate the integral $\int_0^\pi \frac{\cos^2 3\theta}{5 - 4 \cos 2\theta} d\theta$.

2 Attempt any two parts of the following : **10×2=20**

(a) Find the Fourier transform of the following function

$f(x) = 1-x^2$, if $|x| \leq 1$ and $f(x) = 0$, if $|x| > 1$

(b) Using Z — transform solve the following difference equation

$Y_{n+2} - (2 \cos \alpha) Y_{n+1} + Y_n = 7^n$ with the conditions that $Y_0 = 5$, $Y_1 = 1$.

(c) State the convolution theorem for Fourier transform. Prove that the Fourier transform of the convolution of the two functions equal to the product of their Fourier transforms.

3 Attempt any two parts of the following : **10×2=20**

(a) Define skewness and kurtosis of a distribution. The first four moments of a distribution are 0, 2.5, 0.7, and 18.71. Find the coefficient of skewness and kurtosis.

(b) Fit a second degree parabola to the following data :

x	1	2	3	4	5	6	7	8	9
y	2	6	7	8	10	11	8	13	5

- (c) Define coefficient of correlation and regression.
If θ is the acute angle between the two lines of regression then prove that

$$\tan \theta = \frac{1-r^2}{r} \frac{\sigma_x \sigma_y}{\sigma_x^2 + \sigma_y^2}$$

where r , σ_x , σ_y have their usual meanings. Give the significance of the formula when $r = 0$ and $r = \pm 1$.

4 Attempt any two parts of the following : $10 \times 2 = 20$

- (a) Derive Newton – Raphson's method to find a root of the equation $f(x) = 0$. Prove that this method has quadratic convergence.
- (b) Apply Newton's divided difference method to obtain an interpolatory polynomial for the following data.

x	3	5	7	9	11	13
$f(x)$	31	51	17	19	90	110

- (c) Obtain Lagrange's Interpolatory for the following data :

x	1	3	5	7	10
$f(x)$	13	31	25	37	101

Find the values of $f(4)$ and $f(8.5)$.

5 Attempt any two parts of the following : **10×2=20**

(a) Solve the following system of linear equations using

Gauss-Seidel method

$$10x + 3y + 7z = 41,$$

$$3x + 20y + 17z = 101,$$

$$x + 19y + 23z = 201,$$

perform three iterations.

(b) State Simpson's three-eighth rule. Using this rule evaluate the following integral

$$\int_0^6 \frac{x}{1+x^5} dx$$

(c) State Runge-Kutta method of fourth order. Using this method find the values of $y(0.2)$, $y(0.4)$ and $y(0.6)$ for the following initial value problem

$$\frac{dy}{dx} = x^3 - y^3 \text{ with condition that } y(0) = 1$$