**(Following Paper ID and Roll No. to be filled in your Answer Book)**

Paper ID : 110302     Roll No.

**B.Tech.**

**(SEM. III) THEORY EXAMINATION. 2015-16**

**DISCRETE STRUCTURES AND GRAPH THEORY**

[Time : 3 hours]        [Total Marks : 100]

### Section-A

1. Attempt **all** parts. All parts carry **equal** marks. Write answers of each section in short.     (10x2=20)

   (a) Define multiset and power set. Determine the power set $A = \{1, 2\}$.

   (b) Show that $[((pq) => r)(\sim p))] => (q=r)$ is tautology or contradication.

   (c) State and prove pigeon hole principle.

   (d) Show that if set A has 3 elements, then we can have 26 symmetric relation on A.

   (e) Prove that $(P \vee Q) \to (P \wedge Q)$ is logically equivalent to $P \leftrightarrow Q$.

(f) How many 4 digit numbers can be formed by using the digits 2, 4, 6, 8 when repetition of digits is allowed.

(g) The converse of a statements is: If a steel rod is stretched, then it has been heated. Write the inverse of the statement.

(h) If a and b are any two elements of group G then prove $(ab)^{-1} = (b^{-1}a^{-1})$.

(i) If $f: A \rightarrow B$ is one-one onto mapping, then prove that $f^{-1}: B \rightarrow A$ will be one-one onto mapping.

(j) Write the following in DNF $(x+y)(x'+y')$.

## Section-B

Attempt **any five** questions.          $(10 \times 5 = 50)$

2. If Dn define the set of all positive odd integers, i.e. $Dn = \{1, 3, 5 \ldots \ldots \}$, then prove with the help of mathematical induction $P(n) : 1 + 3n$ is divisible by 4.

3. Solve the recurrence relation using generating function: $an - 7an - 1 + 10n - 2 = 0$ with $a0 = 3$, $a1 = 3$.

4. Express the following statements using quantifiers and logical connectives.

   (a) Mathematics book that is published in India has a blue cover.

   (b) All animals are mortal. All human being are animal. Therefore, all human being are mortal.

   (c) There exists a mathematics book with a cover that is not blue.

   (d) He eats crackers only if he drinks milk.

   (e) There are mathematics books that are published outside India.

   (f) Not all books have bibliographies.

5. Draw the Haase digram of $[p (a, b, c), \leq]$, (Note: '$\leq$' stands for subset). Find greatest element, least element, minimal element and maximal element.

6. Simplify the following boolean expressions using k map:

   a) $Y=((AB)'+A'+AB)'$

   b) $A'B'C'D'+A'B'C'D+A'B'CD+A'B'B'CD'=A'B'$

7. Let G be the set af all non-zero real number and let $a*b=ab/2$. Show that $(G,*)$ be an abelian group.

8. The following relation on $A=\{1, 2, 3, 4\}$. Dtermine whether the following :

   a)  $R = \{(1,3), (3,1), (1,1), (1,2), (3,3), (4,4)\}$,

   b)  $R = A \times A$

9. If the permutation of the elements of $\{1,2,3,4,5\}$ are given by $a=(1\ 2\ 3)(4\ 5)$ , $b=(1)(2)(3)(4\ 5)$ , $c=(1\ 5\ 2\ 4)(3)$. Find the value of x, if $ax=b$. And also prove that the set $Z4= (0,1,2,3)$ is a commutative ring with respect to the binary modulo operation $+4$ and $*4$.

## Section-C

Attempt **any two** questions.                    $(2\times15=30)$

10. Let L be a bounded distributed lattice, prove if a complement exists, it is unique. Is $D_{12}$ a complemented lattice? Draw the Hasse diagram of $[P\ (a,b,c), \leq\ ]$, (Note: '$\leq$' stands for subset). Find greatest element, least element, minimal element and maximal element.

11. Determine whether each of these functions is a bijection from R to R.

   (a)  $f(x) = x2 + 1$

   (b)  $f(x) = x3$

   (c)  $f(x) = (x2 + 1)/(x2 + 2)$

12. a)  Prove that inverse of each element in a group is unique.

   b)  Show that G=[(1, 2, 4, 5, 7, 8), X9] is cyclic. How many generators are there? What are they?

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