Printed Pages: 7 NAS-301

(Following Paper ID and Roll No. to be filled in your Answer Book)

Paper ID : 2014073 Roll No. | | | | | | | | | |

B. TECH.

Regular Theory Examination,(Odd Sem-III) 2016-17

MATHEMATICS - III

Time : 3 Hours *Max. Marks : 100*

SECTION - A

1. Attempt all parts of this question. Each question carries two marks. (10×2=20)

a) Evaluate $\int_{|z|=\frac{1}{2}} \frac{e^z}{z^2+1} dz$

b) Find the residue of f(z) = cot z at its pole.

c) Find the Z-transform of the sequence $\{a_n\}$.

d) State the convolution theorem for inverse Z-transform.

e) Discuss in brief the types of correlation.

f) What do you understand by measures of Kurtosis, discuss in brief.

g) Define order of convergence for finding out the root of an transcendental equation.

h) For the data [a, f(a)], [a+h, f(a+h)] and [a+2h, f(a+2h)], find $\Delta^2 f(a)$.

i) Define a diagonal system of simultaneous linear algebraic equations.

j) Write the formula for solving the differential equation $\frac{dy}{dx} = f(x,y)$, $y(x_0) = y_0$ by Runge-Kutta fourth order method.

SECTION - B

2. Attempt any three parts of the following:- (3×10=30)

a) Use Calculus of Residue to evaluate the following integral

$$\int_{-\infty}^{\infty} \frac{\cos x}{(x^2+a^2)(x^2+b^2)}\,dx$$

b) Find the Fourier transform of the following function defined for a > 0 by $f(t) = e^{-at^2}$

c) Find the coefficient of correlation (r) and obtain the equation to the lines of regression for the following data:

x	6	2	10	4	8
y	9	11	5	8	7

d) Using method of least squares, derive the normal equation to fit a parabola $y = a + bx + cx^2$ from the following data:

x	2	3	4	5	6
y	14	17	20	24	29

e) Describe Picard's method for solving differential equation and hence solve the differential equation.

$\frac{dy}{dx} = 1 + xy$ upto third approximation, when y(0)=0

SECTION - C

3. Attempt any two parts of the following : (2×5=10)

a) Find the values of C_1 and C_2 such that the function $f(z) = x^2+c_1y^2-2xy + i(c_2x^2 - y^2 + 2xy)$ is analytic. Also find f(z).

b) Find the poles (with its order) and residue at each poles of the following function:

$$f(z)=\frac{1-2z}{z(z-1)(z-2)^2}$$

c) Find the Laurent series expansion of

$$f(z)=\frac{7z-2}{z(z+1)(z+2)}$$ in the region $1<|z+1|<3$

4. Attempt any two parts of the following:- (2×5=10)

a) Find the root of the equation $2x - \log_{10}x = 7$ which lies between 3.5 and 4.0, using method of false position (five iterations only).

b) Using Newton's forward interpolation formula, find a polynomial function for f(x) and hence evaluate f(0.5), from the following data:

x	0	1	2	3	4
$f(x)$	-1	0	13	50	123

c) Using Lagrange's method for interpolation, find y(10) from the following data:

x	5	6	9	11
y	12	13	14	16

5. Attempt any two parts of the following:- (2×5=10)

a) Evaluate the following integral, using Simpson's three - eight rule:

$$\int_0^6 \frac{dx}{1+x^2}$$

Taking 12 intervals.

b) Apply Gauss-Seidal iteration method to solve the following equations (three iterations only)

$$20x + y - 2z = 17$$
$$3x + 20y - z = -18$$
$$2x - 3y + 20z = 25$$

c) Find $f^1(1.1)$ from the following data:

x	1.0	1.2	1.4	1.6	1.8	2.0
$f(x)$	0.0	0.12	0.55	1.29	2.43	4.00

6. Attempt any two parts of the following : (2×5=10)

a) If for two random variables, x and y with same mean, the two regression lines are

$y = ax + b$ and $x = \alpha y + \beta$, then show that $\frac{b}{\beta} = \frac{1-a}{1-\alpha}$

Also find the common mean.

b) The first four moments of a distribution about the value 4 of the variable are -1.5, 17, -30 and 108. Find the moments about the origin.

c) Out of 800 families with 5 children each, how many families would be expected to have

i) Three boys and two girls

ii) At the most two girls.

Assume that probabilities for boys and girls are equal

7. Attempt any two parts of the following:- (2×5=10)

a) Find the inverse Z-transform of

$$Z(z) = \frac{z}{z-1}, \quad |z| > 1$$

b) Find the finite Fourier sine transform of

$$f(x) = x(\pi - x) \text{ in } 0 < x < \pi$$

c) Using Z-transform, solve the following difference equation.

$u_{n+2} + 2u_{n+1} + u_n = n$ with $u_0 = u_1 = 0$