$\square$

## B TECH <br> (SEM III) THEORY EXAMINATION 2018-19 <br> MATHEMATICS-III

Time: 3 Hours
Total Marks:70
Notes: Assume any Missing Data.

## SECTION - A

1. Attempt $\operatorname{ALL}$ parts of the following:
a) The function $f(x)=e^{x}(\cos y+i \sin y)$ is holomorphic or not.
b) Find the residue of $\frac{z^{2}}{(z-1)(z-2)^{2}}$ at pole $z=2$.
c) Formula of Measure of Kurtosis $\beta_{2}=$
d) The first three central moments of a distribution are $0,15,-31$. Find the moment coefficient of skewness.
e) Obtain the function whose first difference is $9 x^{2}+11 x+5$.
f) Find the normal equation of a curve $y=a x+b x^{2}$
g) Let $f(z)=u(r, \theta)+i v(r, \theta)$ be an analytic function. If $u=-r^{3} \sin 3 \theta$, then find $v$.

## SECTION - B

2. Attempt any THREE parts of the following:
$3 \times 7=21$
a) From the following table of values of $x$ and $y$, obtain $\frac{d y}{d x}$ for $x=1.2,2.2,1.6$.

| $x:$ | 1.0 | 1.2 | 1.4 | 1.6 | 1.8 | 2.0 | 2.2 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $y:$ | 2.7183 | 3.3201 | 4.0552 | 4.9530 | 6.0496 | 7.3891 | 9.0250 |

b) Using Runga-Kutta method of fourth order, find $y(0.8)$ correct to 4 decimat places if $\frac{d y}{d x}=y-$ $x^{2}, y(0.6)=1.7379$, taking $h=0.1$.
c) Using complex integration method, evaluate $\int_{0}^{2 \pi} \frac{\cos 2 \theta}{5+4 \cos \theta} d \theta$.
d) The equations of two regression lines, obtained in a correlation analysis of 60 observations are:
$5 x-6 y=24,768 x-100 y=3608$. What is the correlation coefficient? Show that the ratio of coefficient of variability of $x$ to that of $y$ is $\frac{5}{24}$. What is the ratio of variances of $x$ and $y$ ?
e) The pressure of the gas corresponding to various volumes V is measured, given by the following data:

| $\mathrm{V}\left(\mathrm{cm}^{3}\right)$ | 50 | 60 | 70 | 90 | 100 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $\mathrm{P}\left(\mathrm{kg} \mathrm{cm}^{-2}\right)$ | 64.7 | 51.3 | 40.5 | 25.9 | 78 |

## SECTION - C

3. Attempt any TWO parts of the following:
a) Find the unique polynomial $P(x)$ of degree 2 such that: $P(1)=1, P(3)=27, P(4)=64$, use Lagrange method of interpolation.
b) Using Simpson's $3 / 8{ }^{\text {th }}$ rule on integration, evaluate $\int_{0}^{6} \frac{1}{1+x} d x$
c) Expand $\frac{1}{z^{2}-3 z+2}$ in the region $1<|z|<2$.
4. Attempt any TWO parts of the following:
a) If the probability of hitting a target is $10 \%$ and 10 shots are fired independently. What is the probability that the target will be hit at least once?
b) A die is thrown 276 times and the results of these throws are given below:

| No. appeared on the die | 1 | 2 | 3 | 4 | 5 | 6 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Frequency | 40 | 32 | 29 | 59 | 57 | 59 |

Test whether the die is biased or not.[Tabulated value of $\chi^{2}$ at $5 \%$ level of significance for 5 degree of freedom is 11.09]
c) By Residue method, find the inverse Z-transform of $\frac{z}{z^{2}+7 z+10}$
5. Attempt any TWO parts of the following:
a) The following data regarding the heights (y) and weights (x) of 100 college students are given:

$$
\sum x=15000, \sum x^{2}=2272500, \sum y=6800, \sum y^{2}=463025, \sum x y=1022250
$$

b) Solve $x^{3}-5 x+3=0$ by using Regula-Falsi method correct up to four decimal places.
c) From the table, estimate the number of students who obtained marks between 40 and 45 .

| Marks: | $30-40$ | $40-50$ | $50-60$ | $60-70$ | $70-80$ |
| :--- | :--- | :--- | :--- | :--- | :--- |
| No.of Students: | 31 | 42 | 51 | 35 | 31 |

6. Attempt any $\boldsymbol{T} W O$ parts of the following:
a) Find the residue of $f(z)=\frac{z^{3}}{(z-1)^{4}(z-2)(z-3)}$ at its pole and hence evaluate $\int_{C} f(z) d z$, where $C$ is the circle $|z|=5 / 2$
b) Determine the largest Eigen value and corresponding eigen vector of the matrix $A=\left[\begin{array}{ccc}2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2\end{array}\right]$ till three approximation.
c) Verify Cauchy theorem by integrating $e^{i z}$ along the boundary of the triangle with the vertices at the points $1+i,-1+i$ and $-1-i$.
7. Attempt any TWO parts of the following:
a) Use Picard's method to obtain $y$ for $x=0.2$. Given: $\frac{d y}{d x}=x-y$ with initial condition $y=1$ when $x=0$ correct up to four decimal places.
b) In a normal distribution, $31 \%$ of the items are under 45 and $8 \%$ are over 64 . Find the mean and standard deviation of the distribution. It is given that if $f(t)=\frac{1}{\sqrt{2 \pi}} \int_{0}^{t} e^{-\frac{1}{2} x^{2}} d x$ then $f(0.5)=$ $0.19, f(1.4)=0.42$
c) Prove that $h D=-\log (1-\nabla)=\sin h^{-1}(\mu \delta)$
