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## B.TECH

(SEM-III) THEORY EXAMINATION 2019-20 DISCRETE STRUCTURE \& GRAPH THEORY
Time: 3 Hours
Total Marks:100
Note: Attempt all Sections. If require any missing data; then choose suitably.

## SECTION - A

1. Attempt all questions in brief.
a) Let Aand B be sets. Show that $\mathrm{AXB} \neq \mathrm{BXA}$. Under what condition $\mathrm{AXB}=\mathrm{BXA}$ ?
b) Let R be a binary relation on the set of all positive integers such that:
$\mathrm{R}=\{(\mathrm{a}, \mathrm{b}) / \mathrm{a}-\mathrm{b}$ is an odd positive integers $\}$
Is R reflexive? Symmetric? Transitive?
c) Define the Subgroup of a group.
d) Find the total number of squares in a Chess Board.
e) Define Lagrange's theorem. What is the use of the theorem?
f) Define Multiset and Power set. Determine the power set $\mathrm{A}=\{1,2\}$
g) Define a Partial Ordering .
h) What is a binary Search tree? Explain with example.
i) Prove that $(\mathrm{P} \vee \mathrm{Q}) \rightarrow(\mathrm{P} \wedge \mathrm{Q})$ is logically equivalent to $\mathrm{P} \leftrightarrow \mathrm{Q}$.
j) Write short note on : Isomorphism of graphs.

## SECTION - B

2. Attempt any three of the following:
$10 \times 3=30$
a) Let $\mathrm{f}: \mathrm{X} \rightarrow \mathrm{Y}$ and $\mathrm{X}=\mathrm{Y}=\mathrm{R}$, the set of real number. Find $\mathrm{f}^{-1}$ if
(i) $\quad F(x)=x^{2}$
(ii) $\quad \mathrm{F}(\mathrm{x})=(2 \mathrm{x}-1) / 5$
b) Prove that $(\mathrm{R},+, *)$ is a ring with zero divisors, where R is $2 \times 2$ matrix and + and * are usual addition and multiplication operations.
c) Describe the Boolean duality principle. Write the dual of each Boolean equations:
(i) $x+x^{\prime} y=x+y$
(ii) $\quad(x .1)\left(0+x^{\prime}\right)=0$.
d) Determine the value of each of there prefix expressions:
(i) $-* 2 / 933$
(ii) $+-* 335 / \uparrow 232$
e) Solve the recurrence relation:
$a_{n}=3 a_{n-1}+4^{n-1} \quad$, for $n \geq 0 \& a_{0}=1$

## SECTION - C

3. Attempt any one part of the following:
$10 \times 1=10$
a) Prove that a simple graph with n vertices and k components can have at most $(\mathrm{n}-\mathrm{k})(\mathrm{n}-\mathrm{k}+1) / 2$ edges.
b) Prove by using mathematical induction that:

7+77+777+ $\qquad$ +777 . $\qquad$ $.7=7 / 81\left[10^{n+1}-9 n-10\right]$ for every $n € N$.
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4. Attempt any one part of the following:
$10 \times 1=10$
a) Define preorder, inorder and postorder tree traversal. Give an example of preorder, postorder \& inorder traversal of a binary tree of your choice with at least 12 vertices.
b) Let $R$ be a relation on $R$, the set of real numbers, such that $R=\{(x, y)| | x-y \mid<1\}$. Is $R$ an equivalence relation? justify.
5. Attempt any one part of the following:
$10 \times 1=10$
a) Draw the Haase diagram of $[\mathrm{p}(\mathrm{a}, \mathrm{b}, \mathrm{c}), \leq]$, Find greatest element, least element , minimal element \& maximal element.
b) Simplify the following Boolean function using three variables maps:
(a) $f(x, y, z)=\sum(0,1,5,7)$
(b) $f(x, y, z)=\sum(1,2,3,6,7)$
6. Attempt any one part of the following:
$10 \times 1=10$
a) Express this statement using quantifiers:
"Every student in this class has taken some course in every department in the school of mathematical sciences".
b) Solve the recurrence relation by the method of generating function.
$a_{r}-7 a_{r-1}+10 a_{r-2}=0, r \geq 2, \quad$ Given $a_{0}=3$ and $a_{1}=3$.
7. Attempt any one part of the following:
$10 \times 1=10$
a) Let $\left(\mathrm{A},{ }^{*}\right)$ be a monoid such that for every x in $\mathrm{A}, \mathrm{x} * \mathrm{x}=\mathrm{e}$, where e is the identity element. Show that ( $\mathrm{A},{ }^{*}$ ) is an abelian group.
b) Constructed the truth table for the following statements:
(i) $\left(\mathrm{P} \rightarrow \mathrm{Q}^{\prime}\right) \rightarrow \mathrm{P}^{\prime}$
(ii) $\quad \mathrm{P} \leftrightarrow\left(\mathrm{P}^{\prime} \vee \mathrm{Q}^{\prime}\right)$.

