

- (c) (1) Show that the $x(t) = e^{iw_0 t}$ complex exponential signal is periodic.
- (2) Let $x_1(t)$ and $x_2(t)$ be periodic signals with fundamental periods T_1 and T_2 . Under what condition S is the sum $x(t) = x_1(t) + x_2(t)$ periodic.
- (d) Explain the properties of continuous time LTI system.
- (e) Let $x(t) * h_1(t) = f_1(t)$ and $h_1(t) * h_2(t) = f_2(t)$ with LTI system show that $x(t) * f_2(t) = x(t) * \{h_1(t) * h_2(t)\}$
- (f) Consider a sequence $x(n)$
- $$x(n) = 4 - n \quad 0 \leq n \leq 4$$
- $$= 0 \quad \text{otherwise}$$
- Find its discrete time Fourier transform $X(e^{jw})$.

2 Attempt any **four** parts of the following : **5×4=20**

- (a) Find the Fourier transform of

$$x(t) = e^{-at} \quad \forall t \geq 0$$

$$= 0 \quad \forall t < 0$$

- (b) Describe the time domain properties of ideal frequency selective filters.
- (c) Design a band pass filter that has the centre of its pass band at $w = \frac{\pi}{2}$. Zero in its frequency response characteristic at $w = 0$ and $w = \pi$ and its magnitude response is $\frac{1}{\sqrt{2}}$ at $w = \frac{4\pi}{9}$.

- (d) Determine the Fourier transform of the signal

$$\mathbf{x}(n) = \begin{cases} A, & -M \leq n \leq M \\ \mathbf{0}, & \text{elsewhere} \end{cases}$$

- (e) Determine the output $\mathbf{Y}(n)$ of a relaxed linear time-invariant system with impulse response $\mathbf{h}(n) = a^n \mathbf{u}(n)$, $|a| < 1$ when the input is a unit step sequence, that is $\mathbf{x}(n) = \mathbf{u}(n)$.
- (f) Determine the Fourier transform of the function $\mathbf{y}(n) = \mathbf{x}(n) * \mathbf{h}(n)$.

3 Attempt any **two** parts of the following : **10×2=20**

- (a) (i) Show that distribution function

$$F_X(x) = \int_{-\infty}^x f_X(x) dx \quad \text{where } f_X(x) - \infty$$

the density function of random variable \mathbf{x} .

- (ii) A probability density function is given as

$$f_X(x) = a e^{-b|x|} \quad X \text{ is the random variable, } x = -\infty \text{ to } x = \infty. \text{ Determine the relationship between } a \text{ and } b.$$

- (b) A joint density function of the random variables \mathbf{X} and \mathbf{Y} is given as

$$f_{XY}(x, y) = \begin{cases} e^{-(x+y)} & \text{for } x \geq 0, y \geq 0 \\ \mathbf{0} & \text{otherwise} \end{cases}$$

Determine the followings :

- (1) $P(\mathbf{X} < 1)$
- (2) $P(\mathbf{X} > \mathbf{Y})$
- (c) State different properties of probability density function and probability distribution functions.

4 Attempt any **two** parts of the following : **10×2=20**

- (a) State and prove sampling theorem.
(b) Compute the Fourier transform of the following signals :

(1) $x(n) = 2^n u(-n)$

(2) $x(n) = \left(\frac{1}{4}\right)^n u(n+4)$.

- (c) Explain the discrete time processing of continuous time signal ? To achieve this give the Block diagram of a system.

5 Attempt any two parts of the following : **10×2=20**

- (a) Find z-transform and also the frequency response of

$$h(n) = \left(\frac{1}{2}\right) \left[\left(\frac{1}{2}\right)^n + \left(\frac{-1}{4}\right)^n \right] u(n)$$
 locate the zeros

and poles in z - plane.

- (b) Determine the z-transform of the signals and ROC of the following :

(1) $x(n) = na^n u(n)$

(2) $x(n) = (-1)^{n+1} \frac{a^n}{n} u(n-1)$

- (c) Using z-transform find the convolution two signals

$$x_1(n) = \{1, -2, 1\}$$

$$x_2(n) = \begin{cases} 1, & 0 \leq n \leq 5 \\ 0, & \text{elsewhere} \end{cases}$$