Printed Pages : 7

TCS-405

(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 1071 Roll No.

B. Tech.

(SEM. IV) EXAMINATION, 2007-08

THEORY OF AUTOMATA & FORMAL LANGUAGES

Time : 3 Hours] *[Total Marks : 100*

Note : *(1) Attempt **all** questions.*

*(2) All questions carry **equal** marks.*

(3) Be precise in your answer.

1. Attempt any **four** parts of the following : 5×4=20

(a) Prove or disprove that

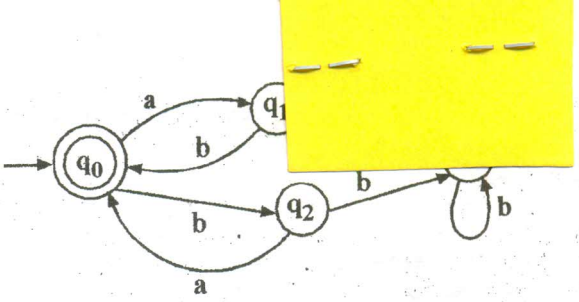
$$(L_1 \cup L_2)^* = L_1^* \cup L_2^*$$

where L_1 and L_2 are languages.

(b) Given a transition diagram for the language L. Show how to build a transition diagram for the language L^*.

(c) Define nondeterministic finite automata. How does it differ from deterministic finite automata ?

(d) Describe in words the language accepted by the following finite automata :



(e) Suppose that the language $L \subseteq \{a, b\}^*$ is defined as follows :

Rule 1 : $a \in L$

Rule 2 : for any $x \in L$, ax is in L.

Rule 3 : for any $x \in L$, xb is in L.

Rule 4 : No other strings are in L.

Describe the language L in terms of set.

(f) Explain the Chomsky classification of languages.

2 Attempt any **two** parts of the following : **10×2**

(a) (i) Design a DFA which accepts the set of strings over alphabet $\sum = \{1, 2, 3, 4\}$ such that string when interpreted as decimal numbers, sum of their digits are divisible by 5.

(ii) Simplify the following regular expressio

$[A]$ $r_1(r_1^* r_1 + r_1^*) + r_1^*$

$[B]$ $(r_1 + r_2 + r_1 r_2 + r_2 r_1)^*$

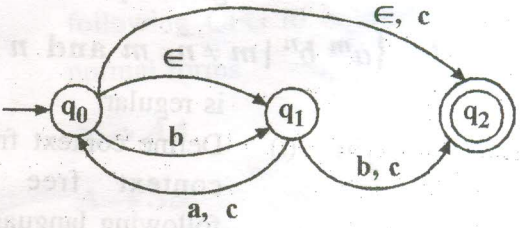
Where r_1 and r_2 are regular expressions

(b) (i) Write regular expression corresponding to the following languages in $\{0, 1\}^*$.

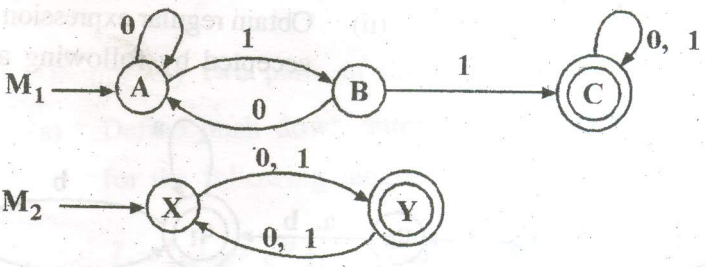
(a) The language of all strings in which every 0 is followed immediately by 11.

(b) The language of all strings that has at most one pair of consecutive 1's.

(ii) Convert following NFA to equivalent DFA and hence minimize the number of states in the DFA.

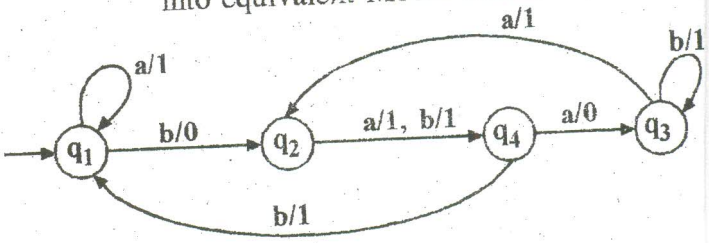


(c) (i) Let M_1 and M_2 be the FA recognizing the languages L_1 and L_2 respectively.



Draw finite automata which recognize the language $L_2 - L_1$.

(ii) Transform the following Mealy machine into equivalent Moore machine.



3 Attempt any **two** parts of the following :

(a) (i) State pumping lemma for regular expression.

(ii) Prove or disprove that the language L given by

$L = \{a^m b^n \mid m \neq n,\ m$ **and** n **are positive integ**

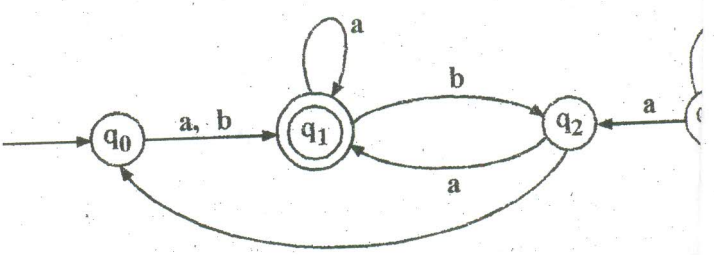
is regular.

(b) (i) Define context free grammer Find a context free grammer for the following language :

$L = \{a^i b^j c^k \mid j \geq i + k;\ i,\ j,\ k$ are

nonnegative integers$\}$

(ii) Obtain regular expression for the languag accepted by following automata :



(c) (i) For the given CFG, find an equivalent CFG with no useless variables

$S \rightarrow AB \mid AC$

$A \rightarrow a\,A\,b \mid b\,A\,a \mid a$

$B \rightarrow bbA \mid aa\,B \mid AB$

$C \rightarrow abCa \mid a\,D\,b$

$D \rightarrow bD \mid a\,C$

(ii) Explain Chomsky normal form and Greibach normal form. Convert the following CFG to equivalent Greibach normal forms

$S \rightarrow AA$

$A \rightarrow SS$

$S \rightarrow a$

$A \rightarrow b$

4 Attempt any **two** parts of the following : **10×2=20**

(a) Define push down automata. Design a PDA for the following language :

$$L = \{a^i\, b^j\, c^k \mid i = j \text{ or } j = k\}.$$

(b) Consider the given PDA :

PDA $M = \left(\{q_0\}, \{0, 1\}, \{a, b, z_0\}, \delta, q_0, z_0, \phi\right)$

where δ is defined as follows

$$\delta(q_0, 0, z_0) = \{(q_0, a\, z_0)\}$$

$$\delta(q_0, 1, z_0) = \{(q_0, b\, z_0)\}$$

$$\delta(q_0, 0, a) = \{q_0, aa\}$$

$$\delta(q_0, 1, b) = \{(q_0, bb)\}$$

$$\delta(q_0, 0, b) = \{(q_0, \in)\}$$

$$\delta(q_0, 1, a) = \{(q_0, \in)\}$$

$$\delta(q_0, \in, z_0) = \{(q_0, \in)\}$$

Convert the given PDA M to the corresponc CFG.

(c) (i) Context free languages are closed under intersection. Prove the statement or give a counter example.

(ii) Given a context free grammer G. Suggest an algorithm which can be used to decide whether language generated by grammer G is empty or not.

5 Attempt any **two** parts of the following : **10×2=20**

(a) (i) Design a turing machine for the following language

$$L = \{ww \mid w \in (a+b)^*\}$$

(ii) Prove that if a language L and its complement both are recursively enumerable then L is recursive.

(b) Design a turing machine which computes function $f : N \rightarrow N$ defined as

$$f(n) = 2^n.$$

(c) Write short notes on the following :

(1) Universal turing machine

(2) Post correspondence problem.