Printed Pages : 7 **TEC – 402**

(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 3082 Roll No. | | | | | | | | | | |

B. Tech.

(SEM. IV) EXAMINATION, 2008-09

SIGNALS AND SYSTEMS

Time : 3 Hours] *[Total Marks : **100***

Note : (i) Attempt **all** the questions.

(ii) All questions carry **equal** marks.

1 Attempt any **four** parts of the following : **5×4=20**

(a) Verify whether

$$g(t) = 10\sin(100\pi t) + 4\cos(200\pi t)$$ is a periodic signal. If yes, determine its fundamental period.

(b) Determine and sketch the spectrum $G(f)$ of the above signal.

(c) Determine the continuous-time unit triangle function ***tri*** **(*t*)** from basic functions.

(d) Determine the continuous-time Fourier transform of the above signal.

(e) The discrete-time Fourier transform of a signal is given by

$$X(F)=\left[\Pi\left(50\left(F-\frac{1}{4}\right)\right)+\Pi\left(50\left(F+\frac{1}{4}\right)\right)\right] * comb(F)$$

where $\Pi(x)=\begin{cases}1; |x|<\frac{1}{2}\\ \frac{1}{2}; |x|=\frac{1}{2}\\ 0; |x|>\frac{1}{2}\end{cases}$ is a unit rectangle function,

$comb(x)=\sum_{n=-\infty}^{n=+\infty}\delta(x-n)$ is the unit *comb* function and '*' denotes the convolution operator. Show that the discrete-time signal $x(n)$ may be given by

$$x(n)=\frac{1}{25}\,\mathrm{sinc}\left(\frac{n}{50}\right)\cos\left(\frac{\pi n}{2}\right)$$

(f) Define a gate function and determine its Fourier transform.

2 Attempt any **four** parts of the following : **5×4=20**

(a) Show that a system with excitation $x(n)$ and response $y(n)$, described by $y(n) = nx(n)$ is linear, time-variant, and static.

(b) Determine the impulse response of the system described by $y(n) + y(n-2) = x(n) - x(n-2)$

(c) A system has an impulse response, $h(t) = 4\exp(-4t)\,u(t)$. Find the response of the system to the excitation

$$x(t) = \Pi\left(2\left(t - \frac{1}{4}\right)\right).$$

(d) Show that the impulse response of the second order continuous-time system described by

$$\frac{d^2y(t)}{dt^2} + 6\frac{dy(t)}{dt} + 4y(t) = x(t)$$ may be given by $h(t) = \exp(-5t)\,u(t)$.

(e) State and prove the Perseval's power theorem.

(f) Determine the fundamental period and frequency of the following discrete-time signals :

(i) $$g(n) = \cos\left(\frac{\pi n}{20}\right)$$

(ii) $$g(n) = \cos\left(\frac{2\pi n}{5}\right) + \cos\left(\frac{2\pi n}{7}\right)$$

3 Attempt any **two** of the following questions :

(a) Obtain the impulse response of the ideal high-pass filter with transfer function given by **10**

$$H_{HP}(f) = H_0\left[1 - \Pi\left(\frac{f}{2B}\right)\right]\exp\left(-j2\pi f\, t_0\right)$$

Where H_0, B and t_0 are constants.

(b) If the Fourier transform of a real signal **10**

$x(t)$ is expressed in terms of its magnitude and phase as

$$X(f) = |X(f)|\exp\left(j\theta(f)\right)$$

Show that

$$|x(f)| = |x(-f)| \quad \text{and} \quad \theta(f) = -\theta(-f)$$

(c) The transfer function of an R-C high-pass system is described by **6+4**

$$H(f) = \frac{j(2\pi RC)f}{1 + j(2\pi RC)f}$$

Determine the impulse response of the system. Under what conditions will it perform as an ideal differentiator defined by the input-out-put relationship

$$y(t) = A\frac{dx(t)}{dt}$$ where A is a constant

4 Attempt any **two** of the following questions :

(a) Let $x(t)$ be a band-limited periodic signal given by $x(t) = 1 + \cos(8\pi t) + \sin(4\pi t)$. Determine and plot the Fourier transform of $x(t)$. If the signal is sampled at twice the Nyquist rate, find the sample values over one fundamental period and find the discrete Fourier transform (DFT) of the sample values. **3+3+4**

(b) Solve the following differential equation by means of Laplace transform : **10**

$$\frac{d^2x(t)}{dt^2} + 6\frac{dx(t)}{dt} + 5x(t) = \exp(-7t)\,u(t)$$

With $x(0) = 0$ and $\left.\frac{dx(t)}{dt}\right|_{t=0} = 0$.

(c) Find the inverse Laplace transform of the following function : 10

$$Y(s) = \frac{s^4 + 5s^3 + 12s^2 + 7s + 15}{(s+2)(s^2+1)^2}$$

5 Attempt any **two** of the following :

(a) Find the Z-transforms of the following discrete-time signals : 3+3+4

(i) $x(n) = \left(\frac{2}{3}\right)^3 u(n+2)$

(ii) $x(n) = \exp(-10n)\, u(n)$

(iii) $x(n) = \exp(n)\sin(n)u(n)$.

(b) Determine the inverse Z-transforms of the following functions : 5+5

(i) $X(z) = \frac{z-1}{z^2 - 4z + 4}$

(ii) $X(z) = \frac{z^2}{z^2 - (5/4)z + 3/8}$.

(c) A discrete-time system with an impulse response **10**

$h(n) = \left(\frac{3}{4}\right)^{|n|}$ is excited by

$$x(n) = \left(\frac{1}{2}\right)^{n} u(n) \left(\frac{2}{3}\right)^{-n} u(-n-1)$$

Find the response of the system.
