

(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 3987Roll No.

--	--	--	--	--	--	--	--	--	--

B.Tech.

(SEM IV) EVEN SEMESTER THEORY EXAMINATION, 2009-2010

MATHEMATICS - III

Time : 3 Hours

Total Marks : 100

- Note :** (i) Attempt **ALL** questions.
(ii) Each question carries **equal** marks.

1. Attempt any four parts of the following : (4x5=20)(a) Show that the function $u = \frac{1}{2} \log (x^2 + y^2)$ is harmonic and find its harmonic.(b) Using Cauchy's integral formula, evaluate $\int_c \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)(z-2)} dz$ where c is $|z|=3$.(c) Expand $f(z) = \frac{1}{(z-1)(z-2)}$ in Laurent's series valid for the regions :(i) $1 < |z| < 2$ (ii) $0 < |z-1| < 1$ (d) Using complex integration method, evaluate $\int_0^{2\pi} \frac{\cos 2\theta}{5 + 4 \cos \theta} d\theta$.(e) Use contour integration method to evaluate $\int_0^{\infty} \frac{x \sin x}{x^2 + a^2} dx, a > 0$.(f) Evaluate $\int_0^{1+i} (x^2 - iy) dz$ along the path $y=x$ and $y=x^2$.

2. Attempt any four parts of the following : (4x5=20)

(a) The equation $f(x) = (x-1)^2 (x-3)^2$ has roots at $x=1$ and $x=3$. Which of the following methods can be applied to find all the roots ?

- (i) Bisection method
- (ii) False - position method
- (iii) Newton - Raphson method

Justify your answer.

- (b) Prove that the Newton Raphson method is second order convergent.
- (c) Perform five iteration of false position method to compute the smallest positive root of the equation $3x + \sin x - e^x = 0$.
- (d) Obtain the value of $f(3.5)$ from the following data :

x	3	4	5	6	7
$f(x)$	3	6.6	15	22	35

- (e) Use Newton's divided difference method to compute $f(5.5)$ from the following data :

x	0	1	4	5	6
$f(x)$	1	14	15	6	3

- (f) Find the missing terms of the following data :

x	1.0	1.5	2.0	2.5	3.0	3.5	4.0
$f(x)$	6	?	10	20	?	15	5

3. Attempt any two parts of the following : (2x10=20)

- (a) Find $f'(1.1)$ and $f''(1.1)$ from the following table :

x	1.0	1.2	1.4	1.6	1.8	2.0
$f(x)$	0.0	0.128	0.554	1.296	2.432	4.000

- (b) Derive the formula for Simpson's $\frac{3}{8}$ rule and find the value of the integral

$$\int_0^1 \frac{dx}{1+x^2}$$

Taking 12 intervals.

- (c) Using Runge - Kutta fourth order method to solve the following differential

equation $\frac{dy}{dx} = \frac{y^2 - x^2}{y^2 + x^2}$ with $y(0) = 1.0$ at $x = 0.2, 0.4$.

4. Attempt any two parts of the following :

(2x10=20)

- (a) Fit a relation $y = ax + \frac{b}{x}$ which satisfies the following data, using method of least squares.

x	1	2	3	4	5	6	7	8
y	5.4	6.2	8.2	10.3	12.6	14.8	17.2	19.5

$$y = an + bn^2$$

$$y = \frac{a}{n} + \frac{b}{n^2}$$

- (b) What do you mean by regression analysis, explain? If for two random variables, x and y with the same mean, the two regression equations are $y = ax + b$ and $x = \alpha y + \beta$

show that $\frac{b}{\beta} = \frac{1 - a}{1 - \alpha}$.

Also find the common mean.

- (c) Let the random variable X assume the value ' n ' with the probability law $p(X = n) = pq^{n-1}$, $n = 1, 2, 3, \dots$

Find the moment generating function and hence mean and variance.

5. Attempt any two parts of the following :

(2x10=20)

- (a) What is chi-square test? A survey of 320 families with 5 children show the following distribution :

Number of boys & girls	5 boys	4 boys	3 boys	2 boys	1 boy	0 boys
	0 girls	1 girl	2 girls	3 girls	4 girls	5 girls
Number of families	18	56	110	88	40	8

(Given $\chi_{0.05}^2 = 11.1$ for 5 d.f.)

Test the hypothesis that male and female births are equally probable.

- (b) Distinguish between the np-chart, p-chart and c-chart of quality control analysis. The average percentage of defectives in 27 samples of size 1500 each was found to be 13.7%. Construct p-chart and give your conclusion regarding quality control.
- (c) In a distribution exactly normal, 7% of the items are under 35 and 89% are under 63. What are the mean and standard deviation of this distribution?

(Note : Ask for the table of area under normal curve from c.s.)