

(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 3987 Roll No. 

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**B. Tech.**

(SEM. IV) THEORY EXAMINATION 2010-11

**MATHEMATICS—III**

Time : 3 Hours

Total Marks : 100

**Note :** (1) Attempt **all** questions.

(2) Provide Chi-Square table.

1. Attempt any **four** parts of the following : (5×4=20)(a) Determine an analytic function  $f(z = u + iv)$ , in terms of  $z$ , whose real part is  $e^{-x}(x \sin y - y \cos y)$ .

(b) Prove that :

(i) 
$$\oint_C \frac{dz}{z-a} = 2\pi i$$

(ii) 
$$\oint_C (z-a)^n dz = 0, n \text{ is an integer } \neq -1,$$

where  $C$  is the circle  $|z-a| = r$ .

(c) State Cauchy's integral formula. Hence evaluate :

$$\int_C \frac{\exp(i\pi z)}{(2z^2 - 5z + 2)} dz$$

where  $C$  is the unit circle with centre at origin and having positive orientation.

(d) Expand  $f(z) = \frac{z}{(z^2 - 1)(z^2 + 4)}$  in Laurent series about in  $1 < |z| < 2$ .

(e) Determine the poles and residues at each poles of the function :

$$f(z) = \frac{z}{z^2 - 3z + 2}$$

and hence evaluate

$$\oint_C f(z) dz, \text{ where } C \text{ is the circle } |z - 2| = \frac{1}{2}.$$

(f) Apply calculus of residues to evaluate

$$\int_0^{2\pi} \frac{d\theta}{5 + 4 \sin \theta}$$

2. Attempt any **four** parts of the following : (5×4=20)

(a) Solve  $\cos x = 3x - 1$  correct to three decimal places using the method of false position.

(b) Find the real root of the equation  $\log_{10} x - x + 3 = 0$  correct to four decimal places using Newton-Raphson method.

(c) Using finite difference method, estimate the missing term in the table :

x :	10	15	20	25	30	35
f(x):	43	-	29	32	-	77

(d) Prove the following relations :

$$(i) \quad \nabla \Delta = \delta^2 = \nabla \Delta \qquad (ii) \quad \nabla E = \Delta = \delta E^{\frac{1}{2}}$$

where symbols have their usual meaning for finite difference.

- (e) Find the Lagrange's interpolating polynomial  $f(x)$  for the following:

$x$	:	0	1	2	5
$f(x)$	:	2	3	12	147

- (f) Estimate the population in 1895 from the following data :
- |                           |   |      |      |      |      |      |
|---------------------------|---|------|------|------|------|------|
| Year                      | : | 1891 | 1901 | 1911 | 1921 | 1931 |
| Population (in thousands) | : | 46   | 66   | 81   | 93   | 101  |

3. Attempt any two parts of the following : (10×2=20)

- (a) Find the value of  $y(1.1)$  using Runge-Kutta method of order four for the differential equation :

$$\frac{dy}{dx} = y^2 + xy, \quad y(1) = 1.0. \quad \text{Take } h = 0.05.$$

- (b) Apply Crout's method to solve :

$$3x - y + 2z = 12$$

$$x + 2y + 3z = 11$$

$$2x - 2y - z = 2.$$

- (c) (i) Find  $f'(s)$  from the following table :

$x$	:	1	2	3	4	5
$f(x)$	:	10	26	50	82	122

- (ii) The velocity ' $v$ ' of a particle at distance ' $s$ ' from a point on its linear path is given in the following table :

$s(m)$	:	0	2.5	5.0	7.5	10.0	12.5	15.0	17.5	20.0
$v(m/sec.)$	:	16	19	21	22	20	17	13	11	9

Apply Simpson's rule to estimate the time taken by the particle to traverse the distance of 20 meters.

4. Attempt any two parts of the following : (10×2=20)

- (a) The first four moments of a distribution about the value 5 of the variable are 2, 20, 40 and 50. Calculate the moments about the mean and comment upon the Skewness and Kurtosis of the distribution.

- (b) Derive the normal equations using the method of least squares to the curve of the form  $y = ax + \frac{b}{x}$ . Hence fit this curve to following data :

x	: 1	2	3	4
y	: -1.5	0.99	3.88	7.66

- (c) (i) If  $\theta$  is the angle between the two regression lines, show that :

$$\tan \theta = \frac{1-r^2}{r} \cdot \frac{\sigma_x \sigma_y}{\sigma_x^2 + \sigma_y^2}$$

Explain the significance when  $r = 0$  and  $r = \pm 1$ .

5. Attempt any two parts of the following : (10×2=20)

- (a) It is given that 2% of the electric bulbs manufactured by a company are defective. Using Poisson distribution, find the probability that a sample of 200 bulbs will contain (i) no defective bulb, (ii) two defective bulbs, (iii) at the most three defective bulbs.
- (b) By using  $\chi^2$  (Chi-Square) test, find out whether there is any association between income level and type of schooling :

Income	Public School	Govt. School
Low	200	400
High	1,000	400

(Given  $\chi^2_{0.05} = 3.84$  for one degree of freedom).

- (c) Discuss how control charts can be used in quality control of industrial products. The average percentage of defectives in 27 samples of size 1,500 each was found to be 13.7%. Construct a suitable control chart for this problem.