Printed Pages—7 **ECS403**

(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 0112 **Roll No.** | | | | | | | | | | |

B. Tech.

(SEM. IV) THEORY EXAMINATION 2010-11

THEORY OF AUTOMATA & FORMAL LANGUAGES

Time : 3 Hours *Total Marks : 100*

Note :— (1) Attempt **ALL** questions.

(2) All questions carry equal marks.

(3) Notations/Symbols/Abbreviations used have usual meaning.

(4) Make suitable assumptions, wherever required.

1. Attempt any **two** parts of the following :

(a) (i) What do you understand by Epsilon-closure of a state in a finite automaton ?

(ii) Minimize the following DFA having state q_5 as final state :

Present State	Next State	
	Input 0	Input 1
$\rightarrow q_0$	q_1	q_2
q_1	q_3	q_4
q_2	q_5	q_6
q_3	q_3	q_4
q_4	q_5	q_6
q_5	q_3	q_4
q_6	q_5	q_6

(iii) Write an algorithm to decide whether the language L(M) accepted by a given finite automata M is infinite ?

(b) (i) Convert the following NFA having r as final state to a DFA :

Present State	Next State			
	a	b	c	$\in$
$\rightarrow$ p	{p}	{q}	{r}	
q	{q}	{r}		{p}
r	{r}		{p}	{q}

(ii) Design a DFA which accepts all those string of a's and b's in which number of a's is even and number of b's is divisible by 3.

(c) State and prove Myhill-Nerode Theorem.

2. Attempt any **four** parts of the following :

(a) Write the regular expression for the following languages :

(i) The set of all strings of 0's and 1's which ends with 1 and does not contain substring 00.

(ii) The set of all strings of 0's and 1's with an equal number of 0's and 1's such that no prefix has two more 0's than 1's nor two more 1's than 0's.

(b) Obtain the NFA without epsilon transition corresponding to the following regular expression :

$$(0^* + 1^*)^* \, 11(1^* \, 0^*)^*.$$

(c) Design a Moore machine with input alphabet {0, 1} and output alphabet {Y, N} which produces Y as output if input sequence contains 1010 as substring, otherwise it produces N as output.

(d) Obtain the regular expression corresponding to the following finite automata with q_0 being start state as well as final states :

Present State	Next State Input a	Next State Input b
$\rightarrow q_0$	q_3	q_0
q_2	q_0	q_3
q_3	q_2	q_4
q_4	q_4	q_2

(e) State whether following statement is true or false. Justify your answer :

(i) If L and M are nonregular langauges then intersection of L and M is also nonregular.

(ii) If L and M are regular languages then L-M is also regular language.

(f) Whether the following language L is regular or not ?

L = $\{0^n \mid n$ is a positive integer and n is not prime$\}$.

Prove your answer.

3. Attempt any **two** parts of the following :

(a) (i) Write a context free grammar for the language L defined as follows :

$L = \{a^i b^j c^k \mid i = j \text{ or } j = k;\ i, j, k \text{ are positive integers}\}$.

(ii) What do you understand by useless symbol in a CFG ? Given the following CFG having S as start symbol, find an equivalent CFG with no useless symbols :

$S \rightarrow AB \mid AC$

$A \rightarrow aAb \mid bAa \mid a$

$B \rightarrow bbA \mid aaB \mid AB$

$C \rightarrow abCa \mid aDb$

$D \rightarrow bD \mid aC$

(b) Consider the following context free grammar G with start symbol S, which generates a set of arithmetic expressions :

$S \rightarrow S + S \mid S * S \mid S \wedge S \mid a$

Given that the precedence of operators in decreasing order is ^, *, +. The operators +, * are left associative while ^ is right associative :

(i) Show that the given grammar is ambiguous.

(ii) Write an equivalent unambiguous context free grammar G_1 which generates the same language.

(iii) If the grammar G_1 has productions of the form $A \rightarrow A\alpha$, $\alpha \in (V \cup T)^*$, write an equivalent context free grammar G_2 which have no production of the form $A \rightarrow A\alpha$.

(c) (i) Write CYK algorithm to decide whether a given string w is member of the language generated by the given context free grammar G or not.

(ii) Convert the following grammar in Greibach normal form :

$$S \rightarrow ASB \mid SS \mid AB$$

$$A \rightarrow 1$$

$$B \rightarrow 1$$

4. Attempt any **two** parts of the following :

(a) (i) Construct a PDA that accepts the language L over {a, b, c} defined as follows :

$L = \{a^m b^n c^p \mid p = m + n;\ m, n \text{ and } p \text{ are integers greater than } 0\}$.

(ii) Consider the context free grammar ({S, A, B}, {a, b}, P, S) where productions P are as follows :

$$S \rightarrow aABB \mid aAA$$

$$A \rightarrow aBB \mid a$$

$$B \rightarrow bBB \mid A$$

Convert the given grammar to PDA that accepts the same language by empty stack.

3. Attempt any t...

(b) Consider the PDA $M = (\{q_0, q_1, q_2\}, \{a, b\}, \{A, Z_0\}, \delta, q_0, Z_0, \Phi)$ where δ is given as follows :

$$\delta(q_0, a, Z_0) = \{(q_0, AZ_0)\}$$

$$\delta(q_0, a, A) = \{(q_0, AA)\}$$

$$\delta(q_0, b, A) = \{(q_1, A)\}$$

$$\delta(q_1, a, A) = \{(q_1, \in)\}$$

$$\delta(q_1, \in, Z_0) = \{(q_2, \in)\}$$

Obtain the context free grammar that generates the same language which is accepted by PDA M.

(c) (i) Define a deterministic push down automata (DPDA). Give an example of a context free language that is not accepted by any DPDA.

(ii) Show that intersection of context free languages may not be context free.

(iii) Given a PDA which accepts language L by empty stack. Suggest procedure for construction of a PDA which accepts L by final state.

5. Attempt any **two** parts of the following :

(a) Define Turing machine. Design a Turing machine that computes the integer function f defined as follows :

$f(n) = 3^n$ where n is integer and $n \geq 0$.

(b) (i) Prove that if a language L and complement of L both are recursively enumerable then L is recursive.

(ii) Give an example of a language that is recursively enumerable but not recursive. Justify your answer.

(c) (i) Define Post Correspondence Problem (PCP) and Modified Post Correspondence Problem (MPCP). Prove that if PCP is decidable then MPCP would be decidable.

(ii) Write short notes on Universal Turing Machine.