EOE048

(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 0934   Roll No.

B.Tech.

(SEMESTER-IV) THEORY EXAMINATION, 2011-12

DISCRETE MATHEMATICS

Time : 3 Hours]   *[Total Marks : 100*

Note :   Attempt questions from each Section as indicated. The symbols have their usual meaning.

Section – A

1.   Attempt **all** parts of this question. Each part carries **2** marks. $10 \times 2 = 20$

(a)   What is Relation ? Define inverse relation with an example.

(b)   What is a Function ? Explain using suitable example.

(c)   Construct the truth table for $(p \wedge q) \wedge r$.

(d)   Show that contrapositives are logically equivalent.

(e)   How many committees of three can be formed from eight people ?

(f)   Find the generating function for the sequence 1, 1, 1, 1, 1, 1.

(g)   Find the number of generators of a cyclic group of order 5.

(h)   Define Coset. What is Right Coset and Left Coset ?

(i)   What is a Graph ? What is Bipartite Graph ?

(j)   Explain Mealy and Moore machines.

Section – B

2. Attempt any **three** parts of this question. $3 \times 10 = 30$

   (a) Prove : If $f : A \to B$ and $g : B \to A$ satisfy $gof = I_A$, then f is one-to-one and g is onto.

   (b) What is a normal subgroup, cyclic subgroup and quotient group ? Explain each with suitable example.

   (c) Find the general solution of $f(n) - 3f(n-1) - 4f(n-2) = 4^n$.

   (d) Prove that the following argument is valid : $p \to \neg q, r \to q, r \vdash \neg p$.

   (e) Prove that for every NDFA, there exists a DFA which simulates the behaviour of NDFA.

Section – C

Attempt **all** questions. **All** questions carry equal marks. $5 \times 10 = 50$

3. Attempt any **two** parts.

   (a) What is reflexive relation ? Consider the following five relations on the set

   $A = \{1, 2, 3, 4\}$

   $\qquad R1 = \{(1,1), (1,2), (2,3), (1,3), (4,4)\}$

   $\qquad R2 = \{(1,1), (1,2), (2,1), (2,2), (3,3), (4,4)\}$

   $\qquad R3 = \{(1,3), (2,1)\}$

   $\qquad R4 = \Phi$, the empty set.

   $\qquad R5 = A \times A$, the universal relation.

   Determine which of the relations are symmetric.

   (b) Explain three pictorial representations of relation on finite set.

   (c) What is recursively defined function ? Explain factorial function.

4. Attempt any **two** parts.

   (a) What is a ring ? What axioms should it satisfy ?

   (b) Let H be a subgroup of G. Prove :

   (i) $H = Ha$ iff $a \in H$.

   (ii) $Ha = Hb$ iff $ab^{-1} \in H$.

   (iii) $HH = H$.

   (c) What is a field ? Prove that a finite integral domain D is a field.

5.　Attempt any **two** parts.

(a)　Prove that a connected graph is Eulerian if it has no vertices of odd degree.

(b)　Draw the binary tree to represent the expression $(x + 3y)^5 (a - 2b)$ and find the expression in preorder notation.

(c)　Draw the transition diagram of a finite – state automation A that accepts the given set of strings over $\{a, b\}$

　　(i)　even number of a's,

　　(ii)　exactly one b.

6.　Attempt any **two** parts.

(a)　What is a tautology ? Verify that the proposition $(p \wedge q) \wedge \neg (p \vee q)$ is tautology or not.

(b)　Show that : (i) $p \wedge q$ logically implies $p \leftrightarrow q$, (ii) $p \leftrightarrow \neg q$ does not logically imply $p \rightarrow q$.

(c)　Show that the propositions $\neg (p \wedge q)$ and $\neg p \vee \neg q$ are logically equivalent.

7.　Attempt any **two** parts.

(a)　Assume there are n distinct pairs of shoes in a coset. Show that if you choose $n + 1$ single shoes at random from the coset, you are certain to have a pair.

(b)　What are generating functions ? Describe its types in brief.

(c)　Find the number of ways in which five large books, four medium size books and three small books can be placed on shelf so that all books of the same size are together.

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