**(Following Paper ID and Roll No. to be filled in your Answer Book)**

**PAPER ID : 3987** **Roll No.** | | | | | | | | | |

**B.Tech.**

**(SEMESTER-IV) THEORY EXAMINATION, 2011-12**

# MATHEMATICS – III

*Time : 3 Hours]* *[Total Marks : 100*

***Note :*** Attempt questions from each Section as indicated. The symbols have their usual meaning. Provide statistical tables which are required by students.

## Section – A

Attempt **all** parts of this question. Each part carries 2 marks : **10 × 2 = 20**

1, (a) If $f(z) = u + iv$ is analytic, then show that the family of curves $u(x, y) = c_1$ and $v(x, y) = c_2$ are mutually orthogonal.

(b) Define removable and essential singular points with example.

(c) Define the coefficients of Skewness and Kurtosis.

(d) What is the total probability theorem ?

(e) Explain in brief Null and Alternative hypotheses.

(f) Define coefficient of contingency.

(g) Isolate the roots of the equation $x^3 - 4x + 1 = 0$.

(h) Verify that $\nabla E \equiv \Delta$.

(i) What do you mean by numerical differentiation ? Explain in brief.

(j) Let $I = \int_{x_0}^{x_3} f(x)dx$, where $f(x)$ is a third degree polynomial. Write the formula you will like to use to find the approximate value of I . It is given that the data are equispaced.

## Section – B

2. Attempt any **three** parts of this question. **3 × 10 = 30**

(a) Verify Cauchy's theorem by integrating exp (iz) along the boundary of the triangle with vertices at the points 1 + i, –1 + i and –1 – i.

(b) Find all four central moments and dicuss skewness and kurtosis and also Karl Pearson skewness for the frequency distribution given in the following table :

| **Range of expenditure (in ₹ 100 per month)** | 2 – 4 | 4 – 6 | 6 – 8 | 8 – 10 | 10 – 12 |
|---|---|---|---|---|---|
| **No. of families** | 38 | 292 | 389 | 212 | 69 |

(c) A manufacturer claimed that at least 95% of the equipments which he supplied to a factory conformed to the specifications. An examination of a sample of 200 pieces of equipments revealed that 18 were faulty. Test this claim at a significant level of (i) 0.05 and (ii) 0.01.

(d) Show that the Newton-Raphson Method has second order convergence.

(e) Solve the following system using Crout's decomposition method :

$$3x - y + 2z = 12$$

$$x + 2y + 3z = 11$$

$$2x - 2y - z = 2$$

## Section – C

**All** questions of this section are compulsory. Attempt any **two** parts from each question. **5 × 10 = 50**

3. (a) Determine p such that the function

$$f(z) = \frac{1}{2}\log(x^2 + y^2) + i\tan^{-1}\left(\frac{px}{y}\right)$$ is an analytic function. Also find f'(z).

(b) Find Laurent series expansion of

$$f(z) = \frac{4z - 1}{z^4 - 1}$$

about the point z = 0.

(c) Evaluate $\int_0^{2\pi} \frac{\cos 2\theta}{5 + 4\cos\theta}\, d\theta$.

4. (a) Fit a parabola of the form $y = a + bx + cx^2$ to the data

| $x$ | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| y | 1.7 | 1.8 | 2.3 | 3.2 |

by the method of least squares.

(b) If $\theta$ is the acute angle between the two regression lines in case of two variables $x$ and y, show that

$$\tan \theta = \frac{1 - r^2}{r} \cdot \frac{\sigma_x \sigma_y}{\sigma_x^2 + \sigma_y^2},$$

where r, $\sigma_x$, $\sigma_y$ have their usual meanings. Explain the significance of the formula when $r = 0$ and $r = \pm 1$.

(c) Out of 800 families with 5 children each, how many would you expect to have (a) 3 boys (b) 5 girls (c) either 2 or 3 boys ? Assume equal probabilities for boys and girls.

5. (a) Fit a binomial distribution to the data given in the following table :

| $x$ | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| f | 24 | 41 | 28 | 5 | 2 |

(b) The number of scooter accidents per month during a year in a certain town were as follows :

12, 8, 20, 2, 14, 10, 15, 6, 9, 4, 7, 13

Are these frequencies in agreement with the belief that the accident conditions were the same during the whole year ?

(c) $\overline{X}$ and R values for 10 sub-groups of 5 readings are given in the following table. Determine the control limits for $\overline{X}$ and R charts for future use, eliminating all the out of control points :

| **Sub-group numbers** | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| $\overline{X}$ | 34.0 | 31.6 | 30.8 | 33.0 | 35.0 | 32.2 | 33.0 |
| **R** | 4 | 4 | 2 | 3 | 5 | 2 | 5 |

| **Sub-group numbers** | 8 | 9 | 10 |
|---|---|---|---|
| $\overline{X}$ | 32.6 | 33.8 | 37.8 |
| **R** | 13 | 19 | 6 |

I Shift. (12)

6. (a) Find the root of $2 \sin x - 2x + 1 = 0$ correct to five significant digits with initial approximation $x_0 = 1.0$.

(b) Estimate the values of a and b in the following table :

| $X$ | 10 | 15 | 20 | 25 | 30 | 35 |
|---|---|---|---|---|---|---|
| f($x$) | 43 | a | 29 | 32 | b | 77 |

(c) The population of a town in decennial census is as under. Estimate the population for the year 1955 :

| Year | 1921 | 1931 | 1941 | 1951 | 1961 |
|---|---|---|---|---|---|
| Population (in lac) | 46 | 66 | 81 | 93 | 101 |

7. (a) Find $\frac{d}{dx}(J_0)$ at $x = 0.1$ from the following table :

| $x$ | 0.0 | 0.1 | 0.2 | 0.3 | 0.4 |
|---|---|---|---|---|---|
| $J_0(x)$ | 1.0000 | 0.9975 | 0.9900 | 0.9776 | 0.9604 |

Also find $\frac{d^2}{dx^2}(J_0)$ at $x = 0.1$.

(b) Evaluate the integral

$$\int_0^{2\pi} e^{-t} \sin(10\, t)\, dt$$

using Simpson's 3/8 rule.

(c) Apply Picard's method to find the solution of the initial value problem

$$\frac{dy}{dx} = y - x, \; y(0) = 2$$

Show that the iterative solution approaches the exact solution.