(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 3987

Roll No. | | | | | | | | | |

B.Tech.

(SEMESTER-IV) THEORY EXAMINATION, 2012-13

MATHEMATICS – III

Time : 3 Hours] *[Total Marks : 100*

Note : Attempt questions from each section as indicated. The symbols have their usual meaning.

SECTION – A

1. **All parts of this question are compulsory :** **10 × 2 = 20**

(a) Find the constants a, b and c such that the function $f(z) = -x^2 + xy + y^2 + i(ax^2 + bxy + y^2)$ is analytic.

(b) Evaluate the integral $\int_C \frac{e^{iz}}{z^3} dz$, where $C : |z| = 1$.

(c) The first-four central moments of a distribution are 0, 2.5, 0.7 and 18.75. Comment on the kurtosis of the distribution.

(d) The equations of two lines of regression are $3x + 12y = 19$ and $9x + 3y = 46$. Find the mean of x and the mean of y.

(e) Enlist the methods by which Trend values can be determined.

(f) Find the moment generating function of Poisson distribution.

(g) Show that $hD \equiv -\sinh^{-1}(\mu\delta)$.

(h) Find the value of $\Delta^2(ab^{cx})$.

(i) Show that $y' = \frac{1}{h}\left[\Delta y - \frac{1}{2}\Delta^2 y + \frac{1}{3}\Delta^3 y - \frac{1}{4}\Delta^4 y + \ldots\right]$

(j) Calculate the value of $\int_4^{5.2} \log_e x \, dx$ by Trapezoidal rule.

SECTION – B

2. Attempt any **three** parts : **3 × 10 = 30**

(a) Using the method of contour integration, evaluate $\int_0^{\infty} \frac{dx}{(x^2 + a^2)^2}$.

(b) Find the multiple linear regression of x_1 on x_2 and x_3 from the data relating to three variables :

x_1	4	6	7	9	13	15
x_2	15	12	8	6	4	3
x_3	30	24	20	14	10	4

(c) In a normal distribution, 31% of the items are under 45 and 8% are over 64. Find the mean and standard deviation of the distribution.

(d) Perform four iterations of the Newton-Raphson method to obtain the approximate value of $(17)^{\frac{1}{3}}$ starting with initial approximation $x_0 = 2$.

(e) Find the value of y(1.1), using Runge-kutta method of fourth order, given that $\frac{dy}{dx} = y^2 + xy$, $y(1) = 1.0$, take $h = 0.05$.

SECTION – C

Note : Attempt any **two** parts from each question. **5 × 10 = 50**

3. (a) Using Cauchy's integral formula, evaluate

$$\int_C \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)(z-3)} dz$$

where C : | z | = 2.

(b) Prove that $\cosh\left(z + \frac{1}{z}\right) = a_0 + \sum_{n=1}^{\infty} a_n \left(z^n + \frac{1}{z^n}\right)$,

where $a_n = \frac{1}{2\pi} \int_0^{2\pi} \text{Cos } n\theta \cdot \text{Cosh } (2 \cos\theta) \, d\theta$.

(c) State and prove Cauchy's Residue Theorem.

4. (a) Find the least squares fit of the form $y = a + bx^2$ to the following data :

x	-1	0	1	2
y	2	5	3	0

(b) Show that the regression co-efficients are independent of the change of origin but not of scale.

(c) Find the moment generating function for triangular distribution defined by

$$f(x) = \begin{cases} x, & 0 \leq x \leq 1 \\ 2-x, & 1 \leq x \leq 2 \end{cases}.$$

5. (a) If the variance of the Poisson distribution is 2, find the probabilities for r = 1, 2, 3 and 4 from the recurrence relation of the Poisson distribution. Also find $P(r \geq 4)$.

(b) Given the following information in the usual notations :

$n_1 = 7, n_2 = 6, S_1^2 = 6.21, S_2^2 = 5.23, \bar{x} = 30$ and $\bar{y} = 28$.

Test the hypothesis that the two samples have come from population having equal means.

(c) 100 students of an engineering institute obtained the following grades in Mathematics paper :

Grade	A	B	C	D	E	Total
Frequency	15	17	30	22	16	100

Using χ^2–test, examine the hypothesis that the distribution of grades is uniform.

6. (a) Find the missing term in the table :

x	2	3	4	5	6
f(x)	45.0	49.2	54.1	?	67.4

(b) Show that the Regula-Falsi Method has linear rate of convergence.

(c) Given the data f(1) = 4, f(2) = 5, f(7) = 5, f(8) = 4. Find the value of f(6) and also the value of x for which f(x) is maximum or minimum.

7. (a) Find the derivative of f(x) at $x = 0.4$ from the following table :

x	0.1	0.2	0.3	0.4
f(x)	1.10517	1.22140	1.34986	1.49182

(b) Use Picard's method to approximate the value of y when $x = 0.1$ given that y= 1 when $x = 0$ and $\frac{dy}{dx} = 3x + y^2$.

(c) Solve the system :

$$x_1 + x_2 + x_3 = 1,$$

$$3x_1 + x_2 - 3x_3 = 5,$$

$$x_1 - 2x_2 - 5x_3 = 10$$

by Crout's method.