(Following Paper ID and Roll No. to be filled in your Answer Books)	
Paper ID : 199414	Roll No.

B.TECH.

Theory Examination (Semester-IV) 2015-16

DISCRETE MATHEMATICS

Time : 3 Hours   *Max. Marks : 100*

Section-A

Q.1. **Attempt all parts. All parts carry equal marks. Write answer of each part in short.**   **(2×10=20)**

(a)   What do you mean by cyclic group explain with example.

(b)   Define Power set and find power set of A= $\{\varnothing, \{\varnothing\}\}$

(c)   What do you mean by Invertible function?

(d)   Distinguish between Tree and Graph.

(e)   Define the absorption and identity law of logic.

(f)   What do you mean by hasse diagram? Draw the hasse diagram of D_{24}.

(1)   P.T.O.

(g) All asymmetric relation is antisymmetric or not. Justify your answer with help of suitable example.

(h) Show that $p \to (p \to q)$ is contingency.

(i) What do you mean by bounded lattice and complete lattice?

(j) Prove that $p \to q \equiv \sim q \to \sim p$

Section-B

Q.2. Attempt any five questions from this section.

$(10 \times 5 = 50)$

(a) Solve the recurrence relation by the method of generating function

$$a_n - 9a_{n-1} + 20a_{n-2} = 0 \quad a_0 = -3, \ a_1 = 1$$

(b) Rewrite the negation of following argument using quantifier variable and predicate symbol

    (i) All birds can fly

    (ii) Some men are genius

    (iii) Some number are not rational

    (iv) There is a student who likes mathematics but not history.

(2)

(c) Show that if $(L, \subseteq, \cup, \cap)$ is a lattice, then $(L, \supseteq, \cap, \cup)$ is also a lattice. Also, show that the Cartesian product of two lattice is a lattice.

(d) Let G be a group and let a, $b \varepsilon$ G be any elements. Then

    (i) $(a^{-1})^{-1} = a$

    (ii) $(ab)^{-1} = a^{-1}b^{-1}$

(e) (i) Let $f : R \to R$ and Let $g : R \to R$, where R is the set of real numbers. find fog and gof, where $f(x) = x^2$ and $g(x) = x+4$. State whether these function are injective, surjective and bijective.

    (ii) If R is an equivalence relation in a set A, then prove that R^{-1} is also an equivalence relation.

(f) State and proof Pigeon hole principle. If there are 15 students in a class then at least how many are born on same day of a week.

(g) Define a distributive lattice. Show that the element of lattice $(N, \leq)$. where N is the set of positive Integer and $a \leq b$ if and only if a divides b. satisfy the distributive property.

(h) Convert the following into CNF

    (a) $\sim(P \vee Q) \leftrightarrow (P \wedge Q)$

    (b) $P \wedge (P \to Q)$

Section-C

Note: Attempt any two questions from this section.

(15×2=30)

Q.3. (a) Prove Lagrange's theorem that states "for any finite group G the order of every group H divides the order of G".

(b) Prove that every cyclic group is an abelian group.

(c) Show that the set [0,1] of all real numbers is not a countable set.

Q.4. Explain the following term with example :

(a) Homomorphism and Isomorphism Graph

(b) Euler Graph and Hamiltonian Graph

(c) Bipartite and Complete Bipartite Graph

Q.5. (a) Prove by principle of mathematical induction that:

P (n):$10^n+3.4^{n+2}+5$ is divisible by 9.

(a) Prove that in a Set A, B, C

(i) $A - (B \cup C) = (A-B) \cap (A-C)$

(ii) $A - (B \cap C) = (A-B) \cup (A-C)$

(b) Construct the truth table for

$p \rightarrow [(p \lor r) \land \sim (p \leftrightarrow \sim r)]$

(4)