Printed Pages: 03 Sub Code: NAS 401

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B. TECH.
(SEM IV) THEORY EXAMINATION 2017-18
MATHEMATICS III

Time: 3 Hours *Total Marks: 100*

Note: Attempt all Sections. If require any missing data; then choose suitably.

SECTION A

1. Attempt *all* questions in brief. 2 x 10 = 20

(a) Determine analytic function $f(z)$ in terms of z whose real part is $x^3 - 3xy^2$.

(b) Write Cauchy's Reimann conditions in polar coordinates system

(c) If the Fourier transform of e^{-x^2} is $\sqrt{\pi}\ e^{-p^2/4}$ then find Fourier transform of $e^{-5(x-2)^2}$.

(d) Find Z transform of a^k.

(e) Write normal equations to fit the curve y = ax^2+b by method of least square.

(f) Find mean and variance of Poisson distribution.

(g) Write the Newton Raphson iteration formula

(h) Prove that third divided difference of $(\frac{1}{a})$ *is* $-\frac{1}{abcd}$.

(i) Discuss diagonal dominant property for system of linear equations.

(j) Use Picard's method to obtain y(0.2) up to two iterations. Given: $\frac{dy}{dx} = x - y$ with the condition y(0)=1

SECTION B

2. Attempt any *three* of the following: 10 x 3 = 30

(a) By contour integration find: $\int_0^\infty \frac{x \sin x}{x^2 + a^2} dx;\ a > 0$.

(b) Using Z-transform solve the difference equation: $y_{k+2} + 4y_{k+1} + 3y_k = 3^k$, given $y_0 = 0,\ y_1 = 1$.

(c) In a normal distribution. 31% of the items are under 45 and 8% are over 64. Find mean and standard deviation of the distribution. It is given that $f(t) = \frac{1}{\sqrt{2\pi}} \int_0^t e^{-\frac{1}{2}x^2} dx$, then $f(0.5) = 0.19$ and $f(1.4) = 0.42$.

(d) Find the real root of equation $3x + \sin x - e^x = 0$ by the method of False-position correct to three decimal places.

(e) Find the value of $y(1.1)$ using Runge-Kutta method of fourth-order, given that $\frac{dy}{dx} = y^2 + xy$, $y(1) = 1$, take $h = 0.05$.

SECTION C

3. **Attempt any *one* part of the following:** **10 x 1 = 10**

(a) State and prove Cauchy's Residue Theorem. Hence or otherwise evaluate $\oint_C \frac{z^2 - 2z}{(z+1)^2(z^2+4)} dz$, where C is $|z| = 10$.

(b) Expand $\frac{7z-2}{z^3 - z^2 - 2z}$ in the regions (i) $1 < |z+1| < 3$ (ii) $0 < |z+1| < 1$(ii) $|z+1| > 3$

4. **Attempt any *one* part of the following:** **10 x 1 = 10**

(a) Find Fourier cosine transform of $\frac{1}{1+x^2}$ and hence find Fourier sine transform of $\frac{x}{1+x^2}$

(b) The temperature u in the semi-infinite rod $0 \le x < \infty$ is determined by the differential equation $\frac{\partial u}{\partial t} = k \frac{\partial^2 u}{\partial x^2}$ subject to the conditions;

(i) u=0 when t=0, $x \ge 0$ (ii) $\frac{\partial u}{\partial x} = -\mu$

(a constant) when x=0 and t>0. Making use of cosine transform, show that

$$u(x,t) = \frac{2\mu}{\pi} \int_0^\infty \frac{\cos px}{p^2} (1 - e^{-kp^2 t}) dp$$

5. **Attempt any *one* part of the following:** **10 x 1 = 10**

(a) The first four moments of a distribution about x = 4 are 1,4,10 and 45. Calculate the moments about the mean and comment upon the skewness and kurtosis of the distribution.

(b) To test the effectiveness of inoculation against cholera, the following table was obtained:

	Attacked	Not attacked	Total
Inoculated	30	160	190
Not inoculated	140	460	600
Total	170	620	790

(The figures represent the number of persons)

Use χ^2-test to defend or refute the statement that that the inoculation prevents attack from cholera. ($\chi_{0.05}^2$ *for* 1 *d.f.* = 3.841).

6. **Attempt any *one* part of the following:** **10 x 1 = 10**

(a) By means of Newton's divided difference formula, find the value of f(15) from the following table:

x	4	5	7	10	11	13
f(x)	48	100	294	900	1210	2028

(b) Estimate from the table, the number of students who obtained marks between 40 and 45.

Marks:	30-40	40-50	50-60	60-70	70-80
No. of students:	31	42	51	35	31

7. **Attempt any *one* part of the following:** **10 x 1 = 10**

(a) Solve by Crout's method, the following system of equations:

$$x+y+z=3,\ 2x-y+3z=16,\ 3x+y-z=-3$$

(b) Find the approximate value of $\int_0^{\frac{\pi}{2}} sinx\, dx$ by Simpson's rule.