Printed Pages : 7 **ECS502**

(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 2165 **Roll No.** | | | | | | | | |

B. Tech.

(SEMESTER-V) THEORY EXAMINATION, 2012-13

DESIGN AND ANALYSIS OF ALGORITHMS

Time : 3 Hours] ***[Total Marks : 100***

Note : Answer **all** the Sections.

Section – A

1. Attempt **all** question parts. **10 × 2 = 20**

(A) Which of the following order of growth is correct ?

(a) $n^2 < n\log_2^n < n!$ (b) $n\log_2^n < n^3 < n!$

(c) $n < \log_2^n < n^2$ (d) $n < 2^n < n^3$

(B) The order of time for creating a heap of size n is

(a) O(n) (b) O(log n)

(c) O (n log n) (d) $O(n^2)$

(C) Quick sort exhibits its worst case behaviour when the input data is in _______ order.

(a) already sorted (b) reverse sorted

(c) random (d) do not have worst case

(D) Every internal node in a B-tree of minimum degree 2 can have ________ children.

(a) 2, 3 or 4 (b) 1, 2 or 3

(c) 2, 4 or 6 (d) 0, 2 or 4

(E) The second largest number from a set of n distinct numbers can be found in

(a) O(n) (b) O(1)

(c) $O(n^2)$ (d) O(log n)

(F) Back-Tracking and Branch-and-bound based solutions use ________.

(a) Spanning Tree (b) Decision Tree

(c) Binary Tree (d) State-space Tree

(G) A function t(n) is said to be in O(g(n)) if t(n)

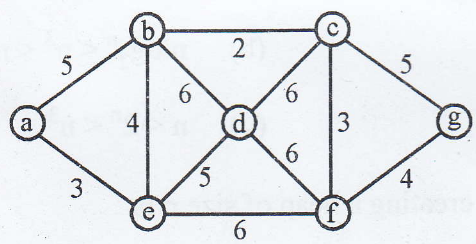
(a) is bounded both above and below by some constant multiples of g(n)

(b) is bounded above by some constant multiple of g(n) for all n

(c) is bounded below by some constant multiple of g(n) for all large n

(d) is bounded above by some function of g(n)

(H) Consider the following graph. Which of the following is NOT the sequence of edges added to the minimum spanning tree using Kruskal's algorithm ?



(a) (b, e), (e, f), (a, c), (b, c), (f, g), (c, d)

(b) (b, e), (e, f), (a, c), (f, g), (b, c), (c, d)

(c) (b, e), (a, c), (e, f), (b, c), (f, g), (c, d)

(d) (b, e), (e, f), (b, c), (a, c), (f, g), (c, d)

(I) A Hamiltonian circuit is

(a) the shortest cycle through all vertices of a graph.

(b) the fastest cycle through distinct vertices of a graph.

(c) a cycle that passes through all the vertices of a graph exactly once excepts the start node.

(d) cycle through points which form the smallest polygon that contains all points of a set of points.

(J) NP is the class of all decision problems whose randomly guessed solutions can be verified in

(a) Deterministic polynomial time

(b) Nondeterministic polynomial time

(c) NP hard time

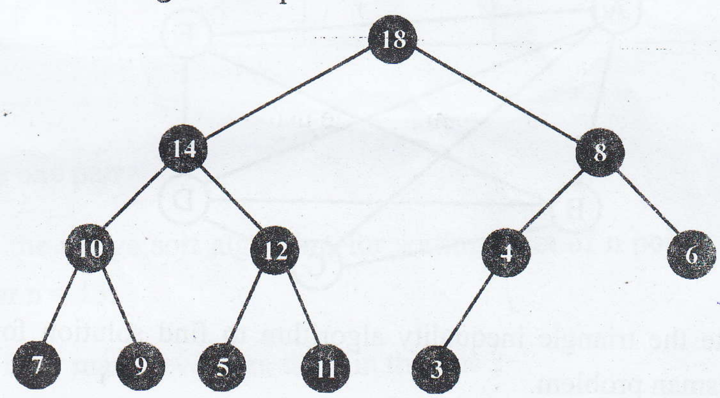
(d) NP complete time

Section – B

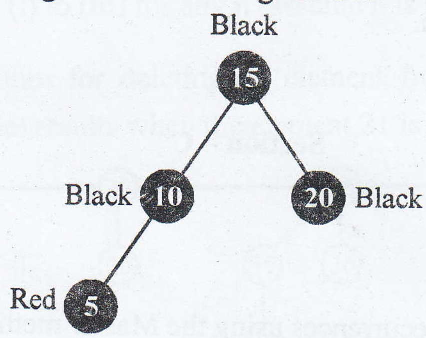
2. Attempt any **three** question parts. **10 × 3 = 30**

(a) (i) Describe the difference between **average-case** and **worst-case** analysis of algorithms, and give an example of an algorithm whose average-case running time is different from its worst-case running time. **(5)**

(ii) How will you represent a max-heap sequentially ? Explain with an example in the below given heap. **(5)**



(b) (i) Consider the following valid red-black tree, where "R" indicates a red node, and "B" indicates a black node. Note that the black dummy sentinel leaf nodes are not shown. Show the resulting red-black tree after inserting key 3 into and deleting 15 from the original tree. **(6)**



(ii) Show any two legal B-Trees of minimum degree 3 that represent {1, 2, 3, 4, 5, 6}. **(4)**

(c) (i) Suppose that undirected graph G = (V; E) has non-negative edge weights and these are raised by 1. Can the minimum spanning tree change ? Can shortest paths change ? Justify with proper example. **(5)**

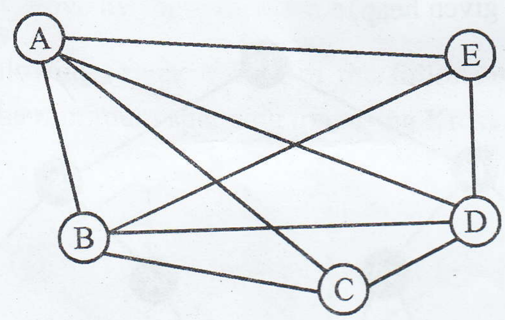
(ii) Show all the steps of Strassen's matrix multiplication algorithm to multiply the following matrices. **(5)**

$$X = \begin{bmatrix} 3 & 2 \\ 4 & 8 \end{bmatrix} \text{ and } Y = \begin{bmatrix} 1 & 5 \\ 9 & 6 \end{bmatrix}$$

(d) (i) Consider the sum-of-subset problem, n = 4, Sum = 13, and $wt_1 = 3$, $wt_2 = 4$, $wt_3 = 5$ and $wt_4 = 6$. Find a solution to the problem using backtracking. Show the state-space tree leading to the solution. Also number the nodes in the tree in the order of recursion calls. **(6)**

(ii) State the implicit and explicit constraints of n-queens problem. **(4)**

(e) In the graph given below : **(10)**



(i) Write the triangle inequality algorithm to find solution for the Travelling Salesman problem.

(ii) Is the solution obtained from the algorithm optimal in all cases ?

(iii) For the graph given above, apply the algorithm starting from city A and obtain the solution. Properly indicate all the intermediate steps of execution of the algorithm.

Section – C

Attempt **all** questions : **10 × 5 = 50**

3. Attempt any **two** parts : **(5 × 2 = 10)**

(a) Solve the following recurrences using the Master method :

$T(1) = 0$

$T(n) = 9\,T(n/3) + n^3 \log n;\ n > 1$

(b) What is the minimum number of keys in a B-tree of order 32 and height 5 ?

(c) Given the six items in the table below and a knapsack with weight limit 100, what is the solution to this knapsack problem ?

Item ID	Weight	Value	Value / Weight
A	100	40	0.4
B	50	35	0.7
C	40	20	0.5
D	20	4	0.2
E	10	10	1
F	10	6	0.6

4. Attempt any **one** part : **(10 × 1 = 10)**

(a) Write the merge sort algorithm for sorting a set of n points. Draw the recursion tree for n = 13.

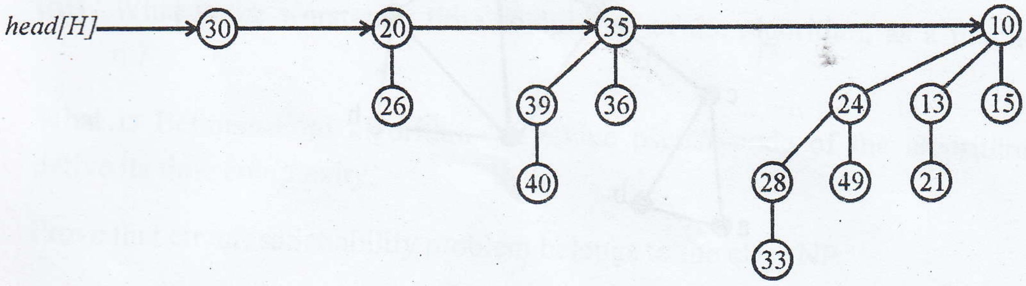
(i) How many levels are there in the tree ?

(ii) How many comparisons are done at each level in the worst case ?

(iii) What is the total number if comparisons needed ?

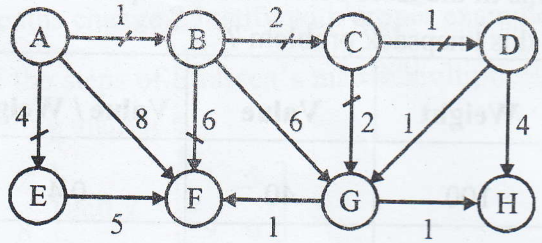
(iv) Generalize (i) to (iii) for any n (assume n is power of 2) in terms of O().

(b) Write the algorithm for deleting an element from a binomial-heap. Show the binomial-heap that results when the element 21 is removed from H given below :



5. Attempt any **one** part : **10 × 1 = 10**

(a) Suppose Dijkstra's algorithm is run on the following graph, starting at node A,



(i) Draw a table showing the intermediate distance values of all the nodes at each iteration of the algorithm.

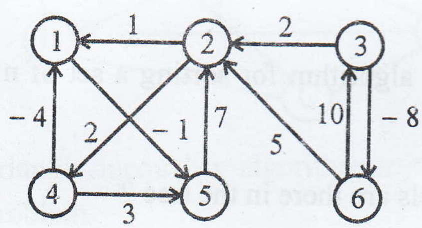
(ii) Show the final shortest path tree.

(b) (i) What is the purpose of Floyd-Warshall's algorithm ?

(ii) Write the pseudo-code of the algorithm.

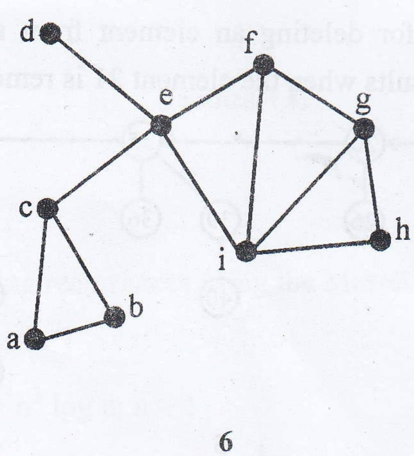
(iii) What is the time complexity of the algorithm.

(iv) Suppose Floyd-Warshall's algorithm is run on the weighted, directed graph shown below, show the values of the matrices that result from each iteration in the algorithm :



6. Attempt any **one** part. **10 × 1 = 10**

(a) Define vertex cover. What is vertex cover problem ? Provide the approximation algorithm for vertex cover problem. Run the algorithm on the graph given below and obtain the solution



(b) Let P = rrllrrll be a pattern and T = lrrrlrrlllrrrlrrlrrllrlrrlrrlr be a text in a string matching problem :

(i) How many shifts (both valid and invalid) will be made by the Naïve String matching algorithm ?

(ii) Provide the algorithm to compute the transition function for a string matching automation.

(iii) Find out the state transition diagram for the automation to accept the pattern P given above.

7. Attempt any **two** parts. **5 × 2 = 10**

(a) You are given the following iterative algorithm :

```
Mystery(A[0, n–1])
//Input: An array A[0, n–1] of n real numbers
for (i = 0; i <= n – 2; i ++) {
for (j = i + 1; j <= n – 1; j++) {
    if (A[i] = = A[j])
        return false;
    }
}
return true;
```

(i) What does this algorithm compute ?

(ii) What is the best-case time complexity of the algorithm, as a function of n ?

(iii) What is the worst-case time complexity of the algorithm, as a function of n ?

(b) What is Bellman-Ford algorithm ? Provide pseudo-code of the algorithm and derive its time complexity.

(c) Prove that circuit satisfiability problem belongs to the class NP.