Printed Pages : 4

ECS-505

(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 110505

Roll No.

B. Tech.

(SEM. V) (ODD SEM.) THEORY EXAMINATION, 2014-15

GRAPH THEORY

Time : 2 Hours] [Total Marks : 50

Note : (1) Attempt **all** questions.

(2) All questions carry **equal** marks.

(3) In case of numerical problems assume data wherever not provided.

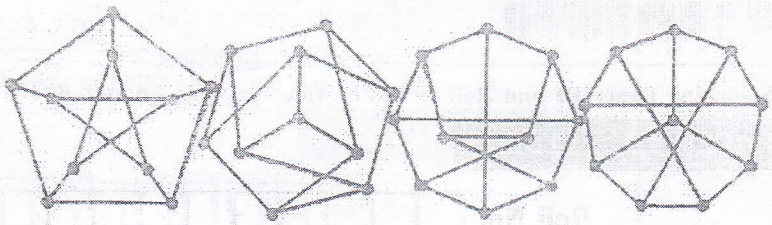
1 Attempt any **three** parts : 3×4=12

(a) Calculate the number of edges in the following graph G if G has

(i) 16 vertices, each of degree 2.

(ii) 3 vertices of degree 4, 2 vertices of degree 3 and other 4 vertices of degree one.

(b) Show that the graphs below are all isomorphic.



(c) Check whether the graphs $K_{3,3}$, $K_{2,4}$, $K_{2,3}$ has

(i) Hamiltonian circuit

(ii) Hamiltonian path

(d) Prove that a connected graph G (with more than one vertex) has an Euler trail if and only if it has exactly two vertices of odd degree. Moreover, the trail originates and ends in the vertices of odd degree.

(e) Prove that the simple graph(connected or disconnected) having n vertices and k Component has at most [(n-k)(n-k+1)]/2 edges.

2 Attempt any **three** parts : **3×4=12**

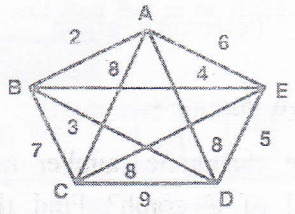
(a) Show that there is no tree with degree sequence

(i) (1,1,2,2,3,3)

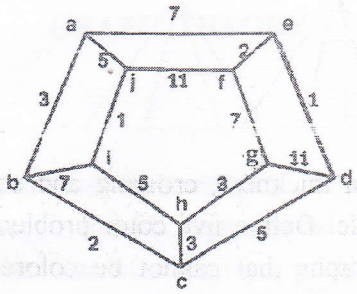
(ii) (1,1,1,1,1,1,2,3,7)

(b) Prove that in a binary tree having n vertices the minimum height is [log2(n+1)-1].

(c) Find a minimal spanning tree for the given connected weighted graph using PRIM's algorithm:



(d) Apply Dijkastra algorithm to find out the shortest path vertices a to d in the following graph.



(e) Explain diameter and radius of a tree with example. Find the condition under which the diameter of a tree is equal to twice the radius.

3 Attempt any **two** parts : 2×6=12

(a) What are geometrical dual and combinational dual graphs. Show that a graph has a dual if and only if it is planar?

(b) Prove that for a planar graph G with n vertices (n>=3), m edges (m>1), and r regions,

(i) n - m + r = 2.

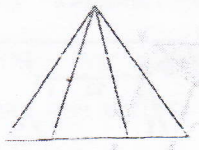
(ii) m >= (3*r)/2

(iii) m <= 3n – 6

(c) State and explain kuratowski theorem and using it show that the Peterson's graph is non-planar. Also find all possible cut set of the Peterson's graph.

4 Attempt any two parts : 2×7=14

(a) Define the chromatic number and chromatic polynomial of a graph? Find the chromatic number and the chromatic polynomial of the following graph



(b) Explain thickness, crossing and covering with example. Define five color problem. Are there any graphs that cannot be colored with four colors?

(c) Define Reduced incidence matrix, Fundamental circuit matrix and Fundamental cut-set matrix of a connected graph? Also devise the relationship between them?